Coherent oscillations in a quantum manifold stabilized by dissipation

S. Touzard$^1$, A. Grimm$^1$, Z. Leghtas$^1$, S.O. Mundhada$^1$, P. Reinhold$^1$, R. Heeres$^1$, C. Axline$^1$, M. Reagor$^1$, K. Chou$^1$, J. Blumoff$^1$, K.M. Sliwa$^1$, S. Shankar$^1$, L. Frunzio$^1$, R.J. Schoelkopf$^1$, M. Mirrahimi$^{2,3}$ & M.H. Devoret$^1$

$^1$Departments of Physics and Applied Physics, Yale University, New Haven, Connecticut 06510, USA.

$^2$Yale Quantum Institute, Yale University, New Haven, Connecticut 06520, USA

$^3$QUANTIC team, INRIA de Paris, 2 Rue Simone Iff, 75012 Paris, France.

The quantum Zeno effect (QZE) is the apparent freezing of a quantum system in one state under the influence of a continuous observation$^{1-3}$. It has been further generalized to the stabilization of a manifold spanned by multiple quantum states$^4$. In that case, motion inside the manifold can subsist and can even be driven by the combination of a dissipative stabilization and an external force$^{5-10}$. A superconducting microwave cavity that exchanges pairs of photons with its environments constitutes an example of a system which displays a stabilized manifold spanned by Schrödinger cat states$^{11-13}$. For this driven-dissipative system, the quantum Zeno stabilization transforms a simple linear drive into photon number parity oscillations within the stable cat state manifold$^{12}$. Without this stabilization, the linear drive would trivially displace the oscillator state and push it outside of the manifold. However, the observation of this effect is experimentally challenging. On one hand, the adiabaticity
condition requires the oscillations to be slow compared to the manifold stabilization rate. On the other hand, the oscillations have to be fast compared with the coherence timescales within the stabilized manifold. Here, we implement the stabilization of a manifold spanned by Schrödinger cat states at a rate that exceeds the main source of decoherence by two orders of magnitude, and we show Zeno-driven coherent oscillations within this manifold. While related driven manifold dynamics have been proposed and observed, the non-linear dissipation specific to our experiment adds a crucial element: any drift out of the cat state manifold is projected back into it. The coherent oscillations of parity observed in this work are analogous to the Rabi rotation of a qubit protected against phase-flips and are likely to become part of the toolbox in the construction of a fault-tolerant logical qubit.

The quantum Zeno effect is the stabilizing back action that a dissipative environment has on a quantum system. Harnessing this effect is the essence of the design of quantum error correction and quantum computation schemes. An important primitive in such research is stabilizing a manifold of states, which requires a dissipation which is blind to the manifold observables. An example of such primitive is provided by a superconducting microwave cavity, in which a dissipative process that annihilates photons in pairs at rate $\kappa_2$, acting together with a two-photon drive of strength $\epsilon_2$ projects the system onto the manifold spanned by Schrödinger cat states $|C^\pm_{\alpha_\infty}\rangle = \mathcal{N}(|\alpha_\infty\rangle \pm |\alpha_\infty\rangle)$, where $|\alpha_\infty\rangle$ is a coherent state of amplitude $\alpha_\infty = \sqrt{2\epsilon_2/\kappa_2}$ and $\mathcal{N}$ is a normalization factor. Each one of these states has a well defined photon number parity, which is conserved by the engineered dissipation. In this Schrödinger cat states manifold, the displacement operator $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ (where $a$ is the annihilation operator acting on the
harmonic oscillator) has two effects: it changes the photon number parity and changes the amplitude of its component coherent states. The engineered dissipation leaves invariant the change in parity and cancels the change in amplitude (Fig 1h). The net result of this Quantum Zeno Dynamics (QZD) is to continuously vary the parity of Schrödinger cat states.

These parity oscillations actually constitute the basis of an X-gate on a qubit encoded in the protected manifold $|0/1\rangle_P = \mathcal{N}(|\alpha_\infty\rangle \pm |-\alpha_\infty\rangle)$. Encoding quantum information in superpositions of Schrödinger cat states is compatible with quantum error correction (QEC) realized with quantum non-demolition parity measurements $^{19,21}$. Our gate is fundamentally different than previous manipulations of Schrödinger cat states $^{22}$ as it operates while the manifold is stabilized. Thus, the quantum information is protected from out-of-manifold gate errors. Moreover the operation of the gate is not affected by the dominant source of errors.

In our experiment, schematically shown in Fig 1, the drive-dissipation is implemented using two-photon transitions between two electromagnetic modes. The first one has a high quality factor and stores the Schrödinger cat states. We refer to it as the storage (subscript S). The second one is used as an engineered cold bath that removes rapidly the entropy from the storage. We refer to it as the reservoir (subscript R). We employ the four-wave mixing capability of a Josephson junction, together with two microwave pumps, to stimulate those transitions. In order to make resonant the conversion from one reservoir photon into two storage photons and vice-versa, the first pump is set at frequency $2f_S - f_R$. The second pump, set at frequency $f_R$, combines with the first one to create pairs of photons in the storage. When the dynamics of the reservoir mode is eliminated, the
density matrix of the storage mode $\rho$ is given by the Lindblad equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_S, \rho] + \frac{\kappa_2}{2} D[a_S^2 - \alpha_\infty^2] \rho,$$

where $H_S$ is a Hamiltonian acting on the storage and $D[L] \rho = 2L \rho L^\dagger - L^\dagger L \rho - \rho L^\dagger L$ is the Lindblad superoperator. As the Lindblad superoperator is engineered to be the dominant term in the dynamics, the dynamical steady states of the system are given by the coherent states $|\pm \alpha_\infty \rangle$. The microwave pumps set the phase and amplitude of the complex amplitude $\alpha_\infty$. The Hamiltonian part of the equation contains the self Kerr effect of the storage mode induced by the Josephson junction and the linear drive that induces the coherent oscillations: $H_S/\hbar = -\chi_{SS}/2(a_S^\dagger a_S^2) + (\epsilon a_S + h.c.)$.

The frequency of the coherent oscillations is maximum when the phase of the linear drive $\epsilon$ is perpendicular to the phase of the stabilized Schrödinger cat states. Thus, this linear drive displaces the Schrödinger cat state perpendicularly to the stabilization axis while the dissipation continuously projects the system back to the stabilized manifold. If the drive respects the adiabaticity condition $|\epsilon| \ll |\alpha_\infty|^2 \kappa_2$ then the net effect of the linear drive is to induce parity oscillations within the stabilized manifold, at frequency $\Omega = 2\epsilon |\alpha_\infty|^{[2]}$.

The adiabaticity condition sets an upper-bound on the frequency of these oscillations, fixed by the maximum $\kappa_2$ that we can engineer. In order to observe this dynamics, we also need the coherence time of the storage mode to be larger than the period of the oscillations. The architecture we designed was key to engineer both a highly coherent storage mode and a large coupling to the environment. We implement the storage mode into a long-lived post cavity made of aluminium $^{[2]}$(Fig 1b). Its finite lifetime induces two types of errors on a protected qubit encoded in the stabilized manifold. First, in absence of stabilization, the amplitude of the Schrödinger cat states
decays until eventually the two coherent states are no longer distinguishable. This error happens at rate $\kappa_1/2\pi = 1.7$ kHz. Second, when the environment is observed to have absorbed a photon, the projected density matrix of the storage mode suffers a parity jump, which corresponds to a bit-flip error in our encoded qubit. For a stabilized cat state containing $\bar{n} = |\alpha_\infty|^2$ photons on average, they happen at a rate $\bar{n}\kappa_1$. Additionally, the above mentioned Kerr effect would distort the coherent states at rate $\chi_{SS}/2\pi \sim 3$ kHz in absence of stabilization. In order to achieve a fast non-linear dissipation, the storage cavity is coupled to a transmon qubit embedded into a coaxial tunnel whose lifetime is engineered to be much less than that of the storage (317 ns), and we use it as the entropy reservoir to induce the QZE. While the reservoir is efficient to dispose of the entropy of the storage mode, its low lifetime has two impacts on the coherence of the storage. First, we associate the lifetime of our storage cavity to the Purcell effect (usually $\kappa_1/2\pi$ is of order $100$ Hz). Second the finite temperature of the reservoir causes additionnal dephasing of the storage mode (see supplement). However, the direct coupling between the storage and the reservoir modes led to a non-linear dissipation rate of $\kappa_2/2\pi = 176$ kHz. The two orders of magnitude separating $\kappa_1$ and $\kappa_2$ are enough to observe the parity oscillations while respecting the adiabaticity condition.

The storage cavity is also coupled to a very coherent transmon whose coherence times ($T_1 = 70$ µs, $T_2 = 30$ µs) are large compared to the time it takes to perform a parity measurement of the storage cavity using the dispersive coupling (218 ns). We use this transmon qubit to measure the Wigner function of the storage mode.

Our experimental protocol follows a fixed sequence of pulses which contains three parts (Fig 2a). The first step is the initialization of the system in the encoding manifold, which is done
with pulses generated by an optimal control algorithm \(^{22}\). As it involves transient states that are not Schrödinger cat states and that are entangled to the transmon qubit, this method induces errors on the protected encoding that are not corrected by the stabilization. However, it is currently the fastest method available. The second part is the stabilization of the manifold, which is done with or without the rotation drive. Finally, in the third part, the Wigner function of the storage cavity at a given point in phase space is measured\(^ {19,27}\).

We characterized the initialization and the quality of the measurement by taking a full Wigner tomography of the storage cavity initialized in \(|0\rangle_P\) and \(|1\rangle_P\) (Fig 2b). The raw data consisted of single shot parity measurements realized with a parametric amplifier and averaged without any further normalization. The phase locking of the different drives ensured that the stabilization axis of the Schrödinger cat states was aligned with the Wigner representation axis, and that the rotation drive was perpendicular to the stabilization axis\(^ {26}\). The right column illustrates our ability to go from an even/odd parity Schrödinger cat state to a "Yurke-Stoler" cat state\(^ {28}\). The zero value of the Wigner function at the center of phase space shows that these states had no parity. They were generated by a rotation of \(\pi/2\) in the encoding manifold.

In order to investigate the oscillations more closely, we restricted the measurement of the Wigner function to the center of phase space (photon number parity measurement). In Fig 3a we present the time evolution of cat states, initially in the even state. We measured their parity over 50 µs while they were stabilized (Fig 3a). For \(\bar{n} = 2, 3\) and 5 we observed decay time constants of respectively 22 µs, 14 µs and 8 µs. This behaviour arises from the natural single-photon jumps of
the cavity. They correspond to bit-flips within the encoding manifold which eventually destroy the coherence of the encoded qubit. The coherence of the encoded qubit is lost at a rate $2\bar{n}\kappa_1^{24}$. This is close to what was found in the experiment and thus shows that the decoherence is mainly due to bit-flips happening during the stabilization. With the rotation drive turned on, the oscillations of the parity over time are similar to Rabi oscillations for a two-level system. For a drive with strength $\epsilon$ the equivalent Rabi frequency is given by $\Omega = 2\epsilon |\alpha_{\infty}|$. We chose a first drive strength $\epsilon_0$ that gave a single oscillation in parity within the decay time of a Schrödinger cat state with amplitude $\bar{n} = 2$. We then repeated the experiment for drive strengths that were multiples of $\epsilon_0$ and for different amplitudes of the initial cat states. On each panel the frequency of the oscillations increases with the drive strength. By looking at curves that correspond to the same drive strength over different panels (same colour) we see that the frequency of oscillation also increases with the amplitude of the initial state. We obtained theory predictions by numerically integrating the evolution of the density matrix and superimposed them on the data. The parameters of the theory were all provided by the results of independent experiments.

We also present in Fig 3b the frequency of the oscillations as a function of the normalized drive strength (Fig 3b). According to theory, the oscillation frequency should depend linearly on the drive strength. The linear fit for $\bar{n} = 2$ gives $\epsilon_0/2\pi = 7$ kHz. This value, when compared to $\bar{n}\kappa_2$, means that we respected the adiabaticity condition for this drive strength. However, when the drive strength increases, this condition is no longer fulfilled. Subsequently, we predict the oscillation frequencies for $\bar{n} = 3$ and 5 with good agreement. The difference between the prediction and the data indicates that the stabilized cat state might have had a larger amplitude than the one measured.
This is corroborated by the fact that the decoherence timescales for $\hat{n} = 3$ and 5 were lower than predicted.

The second panel of Fig 3b shows the evolution of the normalized decay time constant for different cat state amplitudes as a function of the drive strength. If the gate was infinitely slow, the decay constant would be the same as in the non-driven case. However, when $\epsilon/\epsilon_0$ increases, the oscillations decay faster. This is explained by the fact that the gate does not perfectly respect the adiabaticity condition $|\epsilon| \ll |\alpha_\infty|^2 \kappa_2$. Nevertheless, when the number of photons in the initial state is larger, the decay constant of the oscillations gets closer to the ideal limit: the adiabaticity condition is easier to fulfill for higher number of photons. Although encoding with a Schrödinger cat state of larger amplitude increases the bit-flip rate, given by $2\hat{n}\kappa_1$, it increases the quality of our manipulation.

Finally, we measured the effect of this protected Rabi rotation on an arbitrary state of the encoding manifold. We represent a state of the protected qubit by a vector in the Bloch sphere. Its coordinates were found by measuring the equivalent Pauli operators for this encoding. The effect of the gate is accurately described by its effect on the 6 cardinal points of an octahedron within the Bloch sphere. We present the results for a manifold encoding using $|\alpha_\infty|^2 = 3$ (Fig 4). The octahedron on the left shows the initial state. To illustrate the effect of the gate we chose a specific rotation of $\pi/2$ using a drive strength $\epsilon = 6\epsilon_0$. This corresponded to a gate time of $1.8 \mu s$. We compared the results with those obtained by waiting for the same amount of time without applying a drive, which corresponded to applying the identity.
The next step after manipulating an encoded and protected qubit contained in the stabilized manifold of cat states is to address the fault-tolerance of logical operations. A future version of our experiment, which should be accessible with current techniques, will increase $\kappa_2$ above any coupling to other modes and thus achieve two goals: first, it will improve the gate quality to make it better than a gate on a physical qubit. Second, it will suppress the dephasing due to finite temperature in other modes and thus suppress one remaining decoherence channel. All possible remaining errors would then be equivalent to bit-flip errors, which can be corrected by fault-tolerant joint parity measurements on several cavities.
Figure 1: The quantum Zeno dynamics and its implementation. **a,** Conceptual representation of the experiment. The quantum state of a harmonic oscillator is represented here by a point in a 2D plane (not to be confused with phase space). The dark blue circle represents a cross-section of the Bloch sphere of a two-state manifold in the larger Hilbert space of the oscillator. The quantum Zeno effect observed in our experiment corresponds to motion along the circle. A weak excitation drive is applied to the oscillator and the resulting trajectory has both a component along the circle and out of it. The non-linear dissipation and drive (orange) cancels the movement outside the circle while being blind to the position of the quantum state on the circle. **b,** Schematics of the experimental device. The quantum manifold is stabilized within the Hilbert space of the fundamental mode of an aluminium post cavity (cyan, storage in the text). This resonator is coupled to two Josephson junctions on sapphire (yellow for the reservoir and crimson for the transmon qubit, see text), which are read out by stripline resonators (grey). Three couplers (brown) bring microwave drives into the system and carry signals out of it.
Figure 2:  **Experimental protocol and Wigner tomography result.**  

**a,** Sequence of different drives outlined in fig 1. The storage is either initialized in a cat state $\mathcal{N}(|\alpha_\infty\rangle \pm |\alpha_\infty\rangle)$ with optimal control pulses or in a coherent state $|\pm\alpha_\infty\rangle$ with a displacement $D(\pm\alpha_\infty)$. The drives stabilizing the manifold are turned on in 24 ns (purple and yellow). They are on for a duration $\Delta t$ during which the storage drive (cyan) can be turned on to induce the parity oscillations. The drives are left on for another 500 ns and then turned off in 24 ns after which the Wigner function is measured. **b,** In the left column is the Wigner functions of the storage cavity after initialization in an even or odd cat state ($|\alpha_\infty|^2 = 3$). The right column shows the corresponding Wigner functions after a quarter of an oscillation. The colormaps are averaged raw data of the Wigner function measurement (see text) and the orange circles are cuts along Re($\alpha$)=0. The grey solid lines are theoretical curves corresponding to even or odd cats (left column, lines 1 and 2 respectively) and parityless cats (right column, lines 1 and 2). The only fit parameter in the theory is the renormalization of the amplitude by a factor 0.87 on the left, and 0.65 on the right. These factors account for the fidelity of the parity measurement and the decay of the fringes of the cat states during the stabilization.
Figure 3: **Characterization of the oscillations.** a, Evolution of the measured parity as a function of time. The initial cat states are even, with $\tilde{n} = |\alpha_\infty|^2 = 2, 3, 5$ (circles, squares, diamonds). The storage drive is either off (black markers) or on (coloured markers) with various strengths given in units of a chosen base strength $\epsilon_0$. Simulations are shown as solid lines. The minimum of each experimental curve is emphasized for each drive strength (full marker with black contour). b, A fit of the data gives the frequency $\Omega$ and the time constant $\tau$ of the decaying oscillations. The former is plotted as a function of the relative drive strength $\epsilon/\epsilon_0$ (top panel). The case $\tilde{n}=2$ is fitted with a linear function (dashed line). Based on this, we make predictions for $\tilde{n}=3, 5$ (solid lines). The bottom panel shows the characteristic decay time of the oscillations $\tau$, normalized by the decay time of the non-driven case $\tau_0$. 
Figure 4: **Gate on cardinal points of the Bloch sphere of the protected manifold.** An arbitrary cat state $\mathcal{N} \left( \cos\left(\frac{\theta}{2}\right) |C^+_{\alpha_{\infty}}\rangle + \sin\left(\frac{\theta}{2}\right) e^{i\phi} |C^-_{\alpha_{\infty}}\rangle \right)$ is represented by a point on a sphere. Six initial states are chosen, corresponding to the cardinal points $(\theta, \phi) = (0, 0), (\pi, 0), (\pm \pi/2, 0), (\pi/2, \pm \pi/2)$, with $|\alpha_{\infty}|^2 = 3$, and their equivalent Pauli operators are measured. The markers corresponding to each initial state are respectively red and blue circles, grey up/down triangles, and black up/down triangles. The initial octahedron formed by those points (left) is either transformed under the action of the identity (upper right) or a rotation of $\pi/2$ around the X axis (lower right).
References


26. See supplementary materials for details.


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**Correspondence** Correspondence and requests for materials should be addressed to S. Touzard (email: steven.touzard@yale.edu) and M. H. Devoret (email: michel.devoret@yale.edu).

**Current addresses** Z.L.: Centre Automatique et Systèmes, Mines-ParisTech, PSL Research University, 60, boulevard Saint-Michel, 75006 Paris, France.

R.H.: Quantronics Group, SPEC, CEA, CNRS, Université Paris-Saclay, CEA-Saclay, 91191 Gif-sur-Yvette, France.
M.R.: Rigetti Quantum Computing, 775 Heinz Ave, Berkeley, CA 94710

K.S.: Quantum Circuits, Inc., P.O. Box 205494, New Haven, CT 06520-5494