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Optimized driving of superconducting artificial atoms for improved single-qubit gates

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We employ simultaneous shaping of in-phase and out-of-phase resonant microwave drives to reduce single-qubit gate errors arising from the weak anharmonicity of transmon superconducting artificial atoms. To reduce the effect of higher levels present in the transmon spectrum, we apply Gaussian and derivative-of-Gaussian envelopes to the in-phase and out-of-phase quadratures, respectively, and optimize over their relative amplitude. Using randomized benchmarking, we obtain a minimum average error per gate of 0.007 ± 0.005 using 4-ns-wide pulses, which is limited by decoherence. This simple optimization technique works for multiple transmons coupled to a single microwave resonator in a quantum bus architecture.

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The successful realization of quantum information processing hinges upon the ability to perform high-fidelity control (gates) of quantum two-level systems or qubits. A value of 10^{-4} error per gate (EPG) is typically quoted as the necessary threshold for fault-tolerant quantum computation [1,2], although more recent work [2] using two-dimensional codes place this level closer to 10^{-3}. For any qubit, the optimal achievable gate performance is set by the ratio of gate time to coherence time. However, quantum information processing architectures often approximate qubits by two levels of quantum objects with multilevel structure. Effective single-qubit gates are realized with driving fields resonant with the two-level transition. Leakage outside the qubit subspace and phase shifts induced by the presence of other levels can dominate the effective single-qubit gate error. It is an important practical challenge to minimize these deleterious effects using simple optimization schemes that are also applicable in a multiqubit setting.

Optimization of coherent drives for single-qubit gates in large Hilbert spaces has previously been investigated for ion trap [3] and nuclear magnetic-resonance [4] systems, where systematic phase errors rather than decoherence dominate gate performance. The addition of off-resonant drives and dynamical detuning were shown to reduce the dominant phase error. In superconducting quantum circuits, in contrast, single-qubit gate performance typically has been dominated by decoherence. Until recently, it has been possible to decrease single-qubit gate errors to ∼1% using shorter and shorter coherent drives [5]. However, at gate times of ∼10 ns, the weak anharmonicity in superconducting artificial atoms, such as the transmon [6] or the phase qubit [7] presents an apparent impasse. Furthermore, with recent progress in coupling more qubits [8], performing quantum algorithms [9], and detecting entanglement [10,11], the further improvement of effective single-qubit gates in an ever larger Hilbert space becomes an important next step.

In this Rapid Communication, we implement a simple control protocol to improve single-qubit gates in a circuit QED device with two superconducting transmons. We use pulse shaping of a single coherent drive following the recent theoretical exploration [12] of derivative removal via adiabatic gate (DRAG) to simultaneously suppress leakage and phase errors. We demonstrate the improvement of single-qubit gates on both transmons using the first-order correction in DRAG, by switching from rotations induced by Gaussian-enveloped microwave tones to rotations performed using Gaussian and derivative-of-Gaussian envelopes on two quadratures. We tune up the pulses using a set of calibration rotation experiments with results that agree with the model outlined in Ref. [12]. Randomized benchmarking (RB) is employed to show the reduction of the average single-qubit gate error [5,13,14]. For the shortest gate width of 4 ns, we find an improvement by a factor of ∼15 down to a minimum EPG of 0.007 ± 0.005. This is a factor ∼3 improvement over the lowest EPG attainable using only Gaussian pulses, but more importantly is at the limit imposed by two-level decoherence and the current experimental instrumentation. Independently, the DRAG technique has also been implemented at the University of California at Santa Barbara with phase qubits [15].

The DRAG control technique of Ref. [12] prescribes a simple pulse-shaping protocol for reducing single-qubit gate errors due to the presence of a third level. Neglecting the cavity, which is detuned away from any transitions, the driven three-level system, or qutrit, is described by the Hamiltonian,

$$H = \hbar \sum_{j=1,2} [\omega_{0,j} |j\rangle\langle j| + \mathcal{E}(t)\lambda_j(\sigma_j^+ + \sigma_j^-)], \qquad (1)$$

where $\sigma_j^- = |j-1\rangle\langle j|$ and $\sigma_j^+ = |j\rangle\langle j-1|$ are lowering and raising operators, $\omega_{0,j}$ are transition energies to the ground state, and $\lambda_1 = 1$, $\lambda_2 = \lambda = \Omega_{1,2}/\Omega_{0,1}$, give the relative drive coupling strengths of the $0 \leftrightarrow 1$ and $1 \leftrightarrow 2$ transitions, $\Omega_{0,1}$ and $\Omega_{1,2}$, respectively. We can define the anharmonicity of the system as the difference between the $0 \leftrightarrow 1$ and $1 \leftrightarrow 2$ transition frequencies $\alpha_1 = \omega_{1,2} - \omega_{0,1}$. The drive is only turned on for a fixed gate duration t_g and given by $\mathcal{E}(t) = \mathcal{E}^x(t)\cos(\omega_d t) + \mathcal{E}^y(t)\sin(\omega_d t)$, where $\mathcal{E}^{x,y}$ are independent quadrature controls.

Given a large $|\alpha_1| \gg \omega_{0,1}$, the effective Rabi drive rate $\Omega_{1,2}$ to induce any direct or time-dependent transitions to $|2\rangle$ can be negligible. However, for a system such as the transmon, $|\alpha_1|$ is only ∼3%–5% of $\omega_{0,1}$. There are two specific gate errors that arise due to this reduced anharmonicity. With a non-negligible $\Omega_{1,2}$, it becomes possible for the gate targeting the $0 \leftrightarrow 1$ transition to directly populate $|2\rangle$, leaving the qubit subspace. However, a second and more dominant

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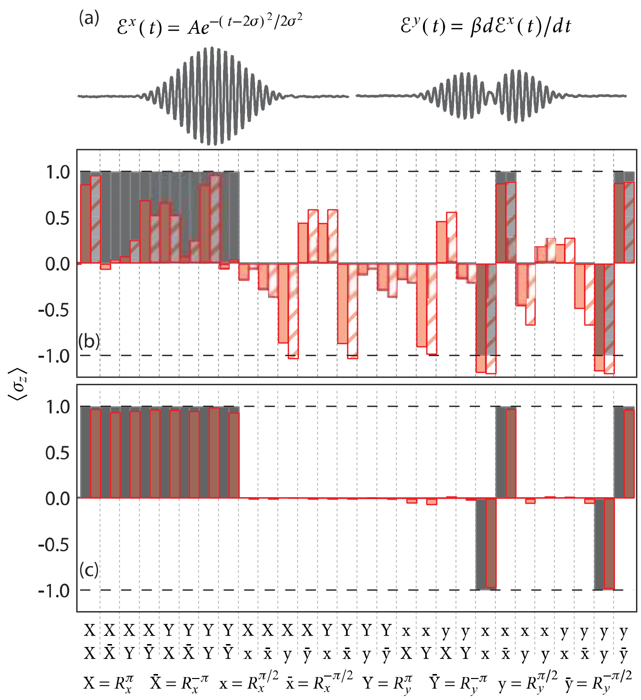

FIG. 1. (Color online) (a) Gaussian and derivative pulse shapes applied to the in-phase and quadrature control channels, respectively, for implementing DRAG to first order. (b) Measured $\langle\sigma_z\rangle$ on transmon L [semitransparent red (lightly shaded) bars], after applying the indicated pairs of π and $\pi/2$ rotations to the ground state. The slash-filled bars correspond to a master-equation simulation of a three-level system with parameters of the sample tested. The gray (darker shaded) bars reflect ideal values. (c) Similarly measured $\langle\sigma_z\rangle$ on transmon L but using DRAG pulses with derivative scale factor $\beta^{\mathrm{L}} = 0.4$ [semitransparent (lightly shaded) bars], also overlaid on the ideal values [gray (darker shaded) bars].

error is a temporary population of $|2\rangle$ during the course of a control pulse to the $0 \leftrightarrow 1$ transition, leading to the addition of a phase rotation to the intended gate. Although Gaussian control pulses [characterized by a standard deviation σ, see Fig. 1(a)] are often the paradigm due to their narrow frequency bandwidth $B = 1/2\pi\sigma$, leakage errors can occur as gate times are reduced such that B is comparable to $|\alpha_1|$. A simplified correction protocol for leakage errors prescribed in Ref. [12] is to apply an additional control on the quadrature channel $\mathcal{E}^y(t) = \beta\dot{\mathcal{E}}^x(t)$ and a dynamical detuning of the drive frequency $\delta(t) = \mathcal{E}^x(t)^2(\lambda^2 + 4\beta\alpha_1)/4\alpha_1\lambda$, where β is a scale parameter. For this paper, we adopt the simplified picture of a qutrit driven without dynamical detuning [16] such that the optimal $\beta = -\lambda^2/4\alpha_1$.

The experiments are performed in a circuit QED sample consisting of two transmons coupled to a coplanar waveguide resonator. The sample fabrication and experimental setup are described in Ref. [9]. The two transmons (designated L and R) are detuned from one another with ground- to excited-state ($0 \leftrightarrow 1$) transition frequencies of $\omega_{0,1}^{\mathrm{L,R}}/2\pi = 8.210, 9.645$ GHz, and the ground-state cavity frequency is $\omega_{\mathrm{C}}/2\pi = 6.902$ GHz. The anharmonicities of the transmons are found using two-tone spectroscopy measurements [17] to be $\alpha_1^{\mathrm{L,R}} = -330, -300$ MHz, and coherence times are measured to be $T_1^{\mathrm{L,R}} = 1.2, 0.9$ μs and $T_2^{*\mathrm{L,R}} = 1.5, 1.1$ μs.

To implement DRAG to first order and perform single-qubit gates on the transmons, we use an arbitrary-wave-form generator (Tektronix 5014, 1 GS/s, 250-MHz bandwidth) to shape microwave-frequency pulses through a vector generator (Agilent E8267D), permitting rotations about either the x or the y axis of each qubit. We fix the drive frequency at $\omega_{0,1}$, and the pulse amplitudes and phases define the rotation angle and axis orientations, respectively. When performing an x rotation, $\mathcal{E}^x(t)$ is a Gaussian pulse shape [Fig. 1(a)], while the derivative of the Gaussian is applied simultaneously on the other quadrature $\mathcal{E}^y(t) = \beta\dot{\mathcal{E}}^x(t)$. All of the pulses are truncated to 2σ from the center, and a buffer time of 5 ns ensures complete separation between concatenated pulses.

A simple test sequence is used to optimize the scale parameter β as well as to demonstrate the effect of using first-order DRAG pulses versus single-quadrature Gaussians. The sequence consists of pairs of π and $\pi/2$ pulses around both the x and the y axes. An important feature of this sequence is that the final average z projection of the single qubit $\langle\sigma_z\rangle$ will ideally take on values from the set $S = \{+1, 0, -1\}$, making any deviations easily visible.

Using Gaussian pulse shaping with $\sigma = 1$ ns and implementing the test sequence for transmon L, we find significant deviations from S, as shown in the solid red (lightly shaded) bars of Fig. 1(b). The theoretical results for each pair of rotations are shown with solid gray (darker shaded) bars in the background. Note that there are some concatenated rotations for which $|\langle\sigma_z\rangle| > 1$, which is because the measurement calibration is performed using a π pulse, which is itself nonideal and plagued by the same errors induced by the third level. We observe that the errors are large and systematic. They are a combination of both leakage and phase rotations.

We repeat the same test sequence but applying the derivative of the Gaussian to the quadrature channel. By varying β, it is possible to find an optimal value such that the measurements of $\langle\sigma_z\rangle$ agree very well with the theoretical predictions. The semitransparent red (lightly shaded) bars of Fig. 1(c) show measured $\langle\sigma_z\rangle$ for transmon L using $\beta = 0.4$. Here, deviations from the ideal gray (darker shaded) bars decrease to $<2\%$. Also, we have applied the DRAG protocol for transmon R, finding the optimal value $\beta = 0.25$ (data not shown). From the experimental determination of β and α_1, we can infer the second excited state coupling strengths $\lambda^{\mathrm{L,R}} = 1.82, 1.41$, ($\sqrt{2}$ without the presence of the cavity). Using λ^{L} and the three-level model of Eq. (1), a master equation simulation for the Gaussian shaping gives the red slash-filled bars in Fig. 1(a), in good agreement with the experiment.

We can understand the deviation of λ from $\sqrt{2}$ due to the cavity modifying the drive strengths $\Omega_{0,1}$ and $\Omega_{1,2}$ via its filtering effect as well as its different coupling to higher levels of the transmon. Specifically, for a transmon in a cavity, we have

$$\Omega_{j-1,j} = \frac{g_{j-1,j}}{\omega_{\mathrm{C}} - \omega_{j-1,j}}, \tag{2}$$

where $j = 1, 2$ for the transmon excitation level and $g_{i,j}$ is the matrix element coupling the $i \leftrightarrow j$ transmon transition to the cavity [6]. Using the relevant parameters of the two transmons in the experiment and including only the fundamental mode of the cavity, we find $\lambda^{L,R} = 1.85, 1.57$, within 12% of those determined from the test sequence. There are other corrections to λ due to the higher modes of the cavity, however, these can be difficult to estimate as a result of cutoff dependence.

We characterize the degree of improvement for single-qubit gates by using the technique of RB [13]. RB allows us to determine the average EPG through the application of long sequences of alternating Clifford gates ($R^{\pi/2}_{x,y}$) and Pauli gates, chosen from $\{\mathbb{1}, R^{\pi}_x, R^{\pi}_y, R^{\pi}_z\}$ [18]. We use the RB pulse sequences originally given in Ref. [13] and adapted to superconducting qubits in Ref. [5] for both the Gaussian and the derivative pulse shaping for transmon L. We truncate the randomized sequences at various lengths and compare the resulting measurement of $\langle\sigma_z\rangle$ to the ideal final state to obtain the gate fidelity $\mathcal{F}$ [5]. There is an exponential decrease in $\mathcal{F}$ with an increasing number of gates in the randomized sequences. This RB protocol is then repeated for various pulse widths $\sigma \in \{1,2,3,4,6\}$ ns.

Using the Gaussian shaping, we find a large reduction in fidelity with the shortest pulses $\sigma = 1$ ns [Fig. 2(a)]. The scattered gray points give $\mathcal{F}$ for 32 different randomized sequences applied as a function of the number of gates in the sequences. When averaged together, we observe a simple decay of $\bar{\mathcal{F}}$ as a function of the number of gates (solid black squares). Fitting the data with an exponential decay (solid black line), we extract an average EPG, EPG $= 1 - \bar{\mathcal{F}}$ of 0.13 ± 0.02. However, when employing the first-order DRAG, we find a dramatic improvement in the gate performance at $\sigma = 1$ ns [Fig. 2(b)]. There is a significant reduction in the spread of the gray points corresponding to all the different randomized sequences, and a fit (solid black line) to the exponential decay of the average fidelity (solid black squares) gives EPG $= 0.007 \pm 0.005$.

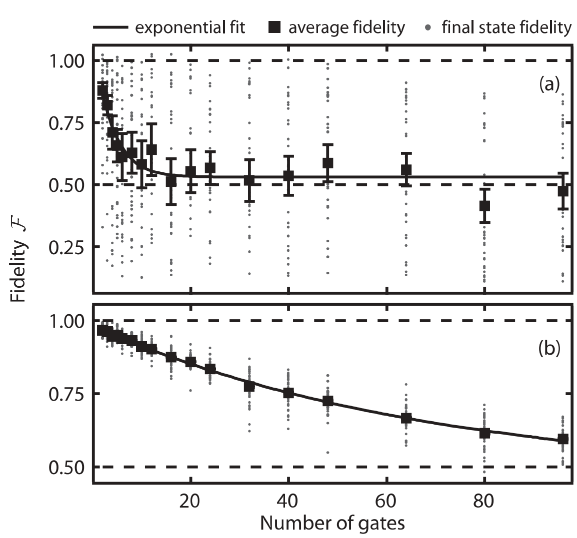

FIG. 2. Randomized benchmarking for transmon L with $\sigma = 1$ ns using (a) Gaussian pulses, and (b) additional Gaussian derivative pulses on the quadrature channel. The scattered gray points are extracted fidelities for 32 RB sequences, truncated at different numbers of gates. A remarkable reduction in the extracted average EPG (black squares) of the benchmarking results is observed going from (a) to (b). The error bars indicate the variance of all the RB sequences and are smaller than the squares in (b).

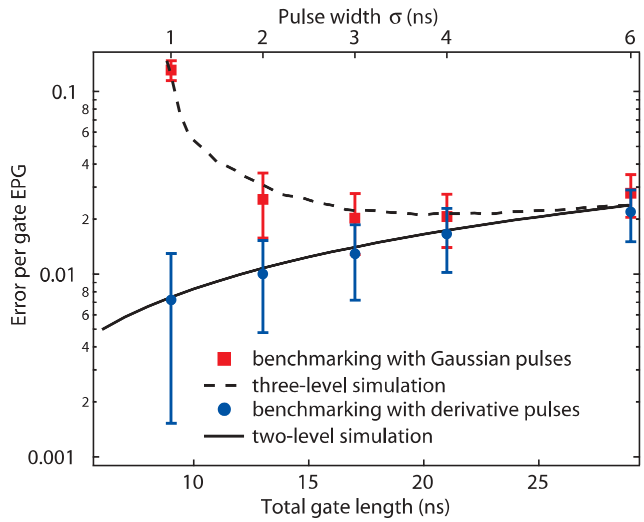

FIG. 3. (Color online) Comparison of single-qubit gate errors with and without DRAG. EPG for the left qubit extracted from RB for different gate lengths using both Gaussian pulses (red squares) and first-order DRAG pulses (blue dots) down to $\sigma = 1$ ns, the shortest permissible by the arbitrary waveform generator. Excellent overlap of the EPG with theory (black solid line) suggests that the DRAG pulses successfully eliminate the errors due to the presence of higher levels. Using DRAG, we reach EPG down to 0.007, which is otherwise unattainable on this sample with Gaussian pulses.

Figure 3 summarizes the improvement to EPG for different σ by using DRAG. The solid squares are the EPG found using Gaussians, revealing a minimum of 0.02 ± 0.007 at $\sigma = 3$ ns, before considerably increasing for shorter pulse lengths. Excellent agreement is found with a master equation simulation (dashed line) of the gate error for a qutrit system incorporating only decoherence times and coupling strengths measured in independent experiments. Using first-order DRAG, we find the solid circles in Fig. 3, which follow a monotonic decrease in EPG with decreasing σ. Here again, we have included a master equation prediction (solid line) of just a single qubit with the same parameters, also giving excellent agreement with the experimentally determined values and demonstrating that DRAG has effectively reduced the response of the transmon to that of a qubit.

Finally, implementing DRAG on both transmons simultaneously, we can produce and detect two-qubit states with high accuracy. Performing state tomography to obtain the two-qubit density matrix ρ via joint readout [11,19] requires 15 linearly independent measurements, corresponding to the application of all combinations of I, R^{π}_x, $R^{\pi/2}_x$, and $R^{\pi/2}_y$ on the two qubits prior to measurement. Thus, errors in these single-qubit rotations applied simultaneously on both qubits can result in incorrect determination of ρ. The two-qubit Pauli set $\vec{P}$ [11] can be used to visualize ρ for the state $|1\rangle_L \otimes |1\rangle_R$ having used Gaussian [Fig. 4(a)] and DRAG [Fig. 4(b)] pulse shaping. $\vec{P}$ consists of ensemble averages of the 15 nontrivial combinations of Pauli operators on both qubits. The ideal $\vec{P}$

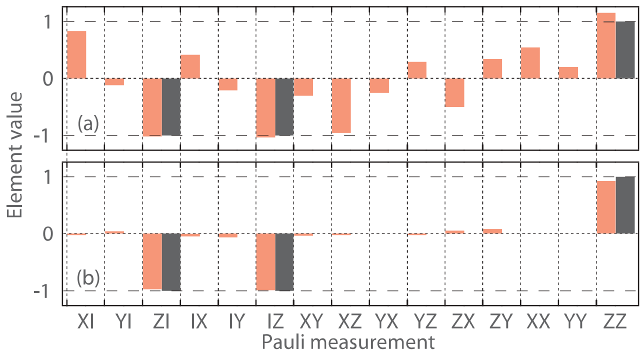

FIG. 4. (Color online) Measured two-qubit Pauli sets for state $|1,1\rangle$ with (a) Gaussian pulses and (b) DRAG pulses ($\beta^{\mathrm{L}} = 0.4$, $\beta^{\mathrm{R}} = 0.25$) applied to the quadrature channels of both transmons L and R. The ideal Pauli set is shown in dark gray.

of the state is characterized by unit magnitude in $\langle ZI\rangle$, $\langle IZ\rangle$, and $\langle ZZ\rangle$ and zero for all other elements. We can see that with the standard Gaussian pulse shaping, there are substantial ($\sim$50%–100% of unity) deviations on ideally zero elements, whereas with the DRAG pulses, the Pauli set bars are very close to their ideal values.

By implementing a simple approximation to the optimal control pulses for a multi-transmon coupled-cavity system, we have reduced gate errors below the 10^{-2} level, limited by decoherence. The agreement of the various experiments with and without DRAG pulse shaping with a qutrit model reflects that gate errors due to the coupling to a higher excited state can be minimized while continuing to shorten gate time. The limitations of DRAG and optimal control can be explored in the future with a tenfold decrease in gate time to approach $\sim$1 ns through improved electronics and a tenfold increase in coherence times to $\sim$10 μs, placing us right at the quoted 10^{-4} fault-tolerant threshold [12]. Furthermore, DRAG is extendable to systems of more than two multilevel atoms for quantum information processing, and has already been employed to enhance single-qubit gates in a circuit QED device with four superconducting qubits [20].

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