Robust readout of bosonic qubits in the dispersive coupling regime

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High-fidelity qubit measurements play a crucial role in quantum computation, communication, and metrology. In recent experiments, it has been shown that readout fidelity may be improved by performing repeated quantum non-demolition (QND) readouts of a qubit’s state through an ancilla. For a qubit encoded in a two-level system, the fidelity of such schemes is limited by the fact that a single error can destroy the information in the qubit. On the other hand, if a bosonic system is used, this fundamental limit could be overcome by utilizing higher levels such that a single error still leaves states distinguishable. In this work, we present a robust readout scheme, applicable to bosonic systems dispersively coupled to an ancilla, which leverages both repeated QND readouts and higher-level encodings to asymptotically suppress the effects of qubit/cavity relaxation and individual measurement infidelity. We calculate the measurement fidelity in terms of general experimental parameters, provide an information-theoretic description of the scheme, and describe its application to several encodings, including cat and binomial codes.

I. INTRODUCTION

The ability to measure a qubit with high fidelity is of great importance in quantum computation [1, 2] and metrology [3, 4], as well as in measurement-based feedback control [5–10] and computation [11–13]. Experimentally, much progress has been made in recent years toward realizing high-fidelity qubit measurement. High-fidelity single-shot measurements have been demonstrated in a wide variety of physical systems, including nitrogen-vacancy centers [14–16], superconducting circuits [17–19], and quantum dots [20–22]. Qubit relaxation is often a limiting factor in such experiments, and in systems with longer qubit lifetimes higher readout fidelities are possible. Indeed, in trapped ions—known for their long coherence times—readout fidelities in excess of 99.9% [23, 24] and even 99.99% [25, 26] have been demonstrated experimentally.

While this experimental progress is encouraging, strategies to further improve qubit readout fidelity are of great interest. One such strategy involves coupling the primary qubit to an ancillary readout qubit. Measurements are performed by mapping the system’s state onto the ancilla, whose state is then read out. These measurements are said to be quantum non-demolition (QND) if the system’s measurement eigenstates are unaffected by the ancilla readout procedure. QND measurements are necessarily repeatable, and the overall measurement fidelity can be improved by repeating measurements to suppress individual measurement infidelity (Fig. 1). Highly QND readouts have already been realized in trapped ion systems [27, 28], nitrogen vacancy centers [9, 29, 30], and circuit QED systems [10, 31–36].

For a qubit encoded in a two-level system, the fidelity of such repeated readout procedures is fundamentally limited by the fact that there exist single errors, such as relaxation of the excited state to the ground state, that can destroy the information in the qubit. This fundamental limit could be overcome, however, by robustly encoding the information within a larger Hilbert space, so that single errors leave states distinguishable. The combination of repeated QND measurements and robust encoding could thus allow one to overcome limits imposed by both individual measurement infidelity and qubit relaxation.

In this work we propose a robust readout scheme for bosonic systems in the dispersive coupling regime—a class of systems where it is possible both to encode information robustly and to read out information in a QND way. The infinite-dimensional Hilbert space of a single bosonic mode (quantum harmonic oscillator) provides room to encode information and protect it from errors [37–39], while the mode’s dispersive coupling to an ancillary quantum system enables repeated QND readout [33, 40–43]. We show explicitly how the combination of these two techniques allows one to simultaneously suppress the contributions to readout infidelity from qubit relaxation and individual measurement noise to higher order, potentially yielding orders-of-magnitude im-

\[
|\psi\rangle = |0\rangle_Z |0\rangle_Z |0\rangle_Z \ldots
\]

FIG. 1. Repeated QND readouts. Contributions to overall measurement infidelity from ancilla preparation/readout errors and noise may be exponentially suppressed by repeating measurements.
improvement in readout fidelity. This scheme is readily applicable to both circuit QED systems [42, 44, 45] and optomechanical systems [46–48], where the dispersive coupling regime is experimentally accessible.

This article is organized as follows. In Sec. II the robust readout scheme is introduced for a lossy bosonic mode coupled to a two-level readout ancilla. We compute the readout infidelity of the scheme for a simple Fock state encoding and show explicitly that contributions from relaxation and individual measurement noise may be suppressed. In Sec. III we consider a bosonic mode subject to both spontaneous relaxation and heating and show how the robust readout scheme may be generalized to also suppress contributions to the infidelity from heating. In Sec. IV we show how, given a readout ancilla with more than two levels, readout fidelity can be significantly improved by using a maximum likelihood estimate as opposed to simple majority voting. Additionally, in this and each of the previous sections, we derive simple approximations for the fidelity that depend on only a small number of general experimental parameters so that readout fidelities may be easily estimated for experimental systems. In Sec. V, we consider the robust readout scheme from the perspective of classical information theory and show how a lower bound may be placed on readout infidelity for given parameters. Finally, in Sec. VI, alternate encodings are considered: we identify general criteria on encodings that are sufficient for robust, ancilla-assisted readout of a qubit encoded in a lossy bosonic mode, show how these criteria are satisfied by cat and binomial codes, and approximate the readout fidelity for both codes.

II. ROBUST READOUT OF A QUBIT ENCODED IN A LOSSY BOSONIC MODE

A. Robust readout scheme

Let a bit of quantum information be encoded in a bosonic mode as \( \ket{\psi}_B = \alpha \ket{0}_B + \beta \ket{1}_B \), where \( \ket{0}_B \) and \( \ket{1}_B \) are the “logical” states in the mode’s Hilbert space that we seek to distinguish with maximal fidelity. Readout of this qubit (henceforth referred to as the bosonic qubit) is performed by repeatedly mapping its state onto a two-level ancillary quantum system, whose state is subsequently measured. The mapping and ancilla readout processes are assumed to be QND so that they may be repeated without disturbing the bosonic qubit. This assumption is justified in the dispersive coupling regime, as will be shown explicitly.

We refer to the process of mapping the bosonic qubit onto the ancilla, followed by ancilla readout, as a level-1 readout. Each level-1 readout yields one classical bit of information (the ancilla is either found to be in \( \ket{g} \) or \( \ket{e} \)). In our scheme, \( N \) repeated level-1 readouts are performed, and their outcomes are collectively analyzed, e.g. with majority voting, to yield a single bit of classical information (the bosonic qubit is determined to be in either \( \ket{0}_B \) or \( \ket{1}_B \)). We refer to the entire procedure—performing \( N \) level-1 readouts and combining the results—as a level-2 readout. This scheme is shown schematically in Fig. 2(a).

![Fig. 2. Robust readout scheme. (a) Quantum circuit for the readout scheme. The state of the bosonic qubit is read out through repeated QND mappings of its state onto an ancilla. (b) The bosonic mode and mapping procedure. Fock states in the bosonic mode decay with rates proportional to their excitation number. All excited states are mapped to the excited state of the two-level readout ancilla.](image)

We now define the logical states, the specific mapping required for this scheme, and the relaxation properties of the bosonic mode, all of which are summarized in Fig. 2(b). The logical states encoding the bosonic qubit are chosen to be the Fock states

\[
\begin{align*}
\ket{0}_B &= \ket{0} \\
\ket{1}_B &= \ket{L},
\end{align*}
\]

for positive integer \( L \). This choice is made to simplify the analysis of readout fidelity. Other possible
choices of the logical states, including cat and binomial codes, are considered in Sec. VI.

In the dispersive coupling regime, these logical states can be distinguished through a measurement procedure that is QND. In this work we consider projective measurements and define QND as follows. A projective measurement can be described by a collection of measurement operators \( \{ \hat{M}_k \} \) that constitute a complete set of orthogonal projectors, satisfying \( \hat{M}_k^2 = \hat{M}_k \) and \( \sum_k \hat{M}_k = 1 \). Such a measurement is QND if

\[
[\hat{M}_k, \hat{H}(t)] = 0,
\]

for all \( k \) and \( t \), where \( \hat{H}(t) \) is an operator describing the ancilla preparation, its coupling to the bosonic mode, and the ancilla readout. In the robust readout scheme, the level-1 measurements are defined by operators \( \hat{M}_0 \) and \( \hat{M}_1 \) that act on the Hilbert space of the bosonic mode

\[
\hat{M}_0 = |0\rangle \langle 0|, \\
\hat{M}_1 = |1\rangle \langle 1| + \ldots + |L\rangle \langle L|.
\]

For a bosonic mode dispersively coupled to a two-level ancilla, QND measurements are possible because these operators commute with the dispersive coupling Hamiltonian,

\[
H_{DC} = -\chi \hat{a}^\dagger \hat{a} \langle e | \langle e |,
\]

where \( |g\rangle \) and \( |e\rangle \) denote the basis states of the ancilla, and \( \hat{a} \) is the bosonic annihilation operator. Similar QND measurements have already been demonstrated experimentally in circuit QED systems \([31]\).

During the mapping process, the bosonic state \( |0\rangle \) (\( |L\rangle \)) is mapped to the ancilla state \( |g\rangle \) (\( |e\rangle \)), while all intermediate Fock states \( |n\rangle \) (\( 0 < n < L \)) are mapped to the ancilla state \( |e\rangle \). In practice, this mapping could be realized in a QND way by initializing the ancilla in the excited state, then using the dispersive coupling to apply a selective pulse \([31, 44, 49, 50]\) that flips the ancilla conditioned on whether the bosonic mode is in \( |0\rangle \).

Because readouts are frequently limited by qubit lifetime, we consider a bosonic mode that is subject to spontaneous relaxation. Specifically, the decay rate of a Fock state \( |n\rangle \) to \( |n-1\rangle \) is given by \( n\kappa_{ij} \), where the factor of \( n \) is due to bosonic enhancement. Transitions between non-adjacent Fock states are suppressed by selection rules, and excitations will be considered later in Sec. III.

As a figure of merit for this readout scheme, the readout fidelity \( \mathcal{F} \) is defined as \([19, 51]\)

\[
\mathcal{F} = 1 - P(0_B|1_B) - P(1_B|0_B),
\]

where \( P(i|j) \) is the probability of the level-2 readout yielding \( i \) when the initial state of the bosonic qubit was \( j \), for \( i, j \in \{0, 1\} \). \( \mathcal{F} \) varies continuously from 0, for readouts which yield no information about the initial state, to 1, for perfect readouts. In the robust readout scheme, both \( P(0_B|1_B) \) and \( P(1_B|0_B) \) may be suppressed by increasing \( L \) and \( N \), as is shown quantitatively in the following sections.

### B. Discrete model of the robust readout scheme

A Hidden Markov Model (HMM) is used to model the robust readout scheme of Fig. 2. A HMM is a Markov chain where, instead of being able to observe a system's state directly, the only information about the system is provided by a series of noisy emissions. HMMs have been previously used as effective models of qubit readout \([52–55]\). In our case, a discrete model (Fig. 3) is used where each level-1 readout is modeled as a possible transition, representing the bosonic qubit's decay, followed by a noisy emission, representing the mapping and the readout of the ancilla.

The model is parameterized by transition probabilities \( T_{ij} \) and emission probabilities \( E_{ij} \). The transition probability \( T_{ij} \) is defined to be the probability that the bosonic state \( |i\rangle \) transitions to \( |j\rangle \) during a single level-1 readout, with \( i, j \in \{0, 1, \ldots, L\} \). The emission probability \( E_{ij} \) is the probability that the bosonic system, having transitioned to state \( |i\rangle \), with \( i, \in \{0, 1, \ldots, L\} \), is read out as ancilla state \( |g\rangle \) for \( j = 0 \), or \( |e\rangle \) for \( j = 1 \). The emission probabilities may be defined in terms of the probability \( \delta \) that an error occurs during the mapping and readout processes which causes the ancilla readout to be misleading

\[
E_{ij} = \begin{cases} 1 - \delta, & \text{if } i = j = 0 \text{ or } i > 0, j = 1, \\ \delta, & \text{otherwise}. \end{cases}
\]

In cases where different Fock states have different probabilities of producing misleading ancilla readouts, \( \delta \) could be chosen as the largest of these probabilities to be conservative.

Explicit expressions for transition probabilities \( T_{ij} \) are derived from the bosonic decay rates. Consider a population of quantum harmonic oscillators, with \( p_i(t) \) of the oscillators in Fock state \( |i\rangle \) at time \( t \). The system of differential equations describing the time evolution of the populations is

\[
\dot{p}_i(t) = \sum_{j=0}^L (K_i)_{ij} p_j(t),
\]
Because \( \tau_0 \) grows with \( L \), so too does the effective signal lifetime, thereby improving readout fidelity. Indeed, the effective lifetime diverges with \( L \), though there are diminishing returns in using higher levels because the divergence is only logarithmic. Interestingly, it should be noted that using higher-level encodings can improve readout fidelity even in the absence of an increase in effective signal lifetime [56].

C. Readout infidelity in the discrete model

The infidelity \( 1 - \mathcal{F} \) of the robust readout scheme may be calculated with the HMM, for given \( L \) and \( N \), in terms of the “experimental” parameters \( \delta \) and \( \kappa_j \tau \). This infidelity depends on how the level-2 measurement outcomes are determined. We consider two approaches: simple majority voting and a maximum likelihood estimate (MLE).

In majority voting, each level-2 measurement outcome is determined by tallying the \( N \) level-1 measurement outcomes, with ancilla readouts of \( |0\rangle \) \((|1\rangle)\) counted as votes for initial state \( |0\rangle \) \((|1\rangle)\). In the MLE, which is the statistically optimal approach, the known values of the transition and emission matrix elements are used to calculate which initial state was more likely to have produced a series of observed ancilla readouts. Explicitly, the likelihood \( \lambda_a(i) \) that a discrete set of ancilla readouts \( a_n \in \{0,1\} \), for \( n \in \{1, \ldots, N\} \), was produced with initial state \( |i\rangle \) is

\[
\lambda_a(i) = \sum_{j_1, \ldots, j_N} T_{i,j_1} E_{j_1,a_1} \cdots T_{j_{N-1},j_N} E_{j_N,a_N}, \tag{11}
\]

which may be calculated efficiently in \( O(NL^2) \) operations [57]. The outcome of a level-2 measurement is then decided by determining which of the two initial states was more likely to have produced the emissions, i.e. by comparing \( \lambda_a(0) \) and \( \lambda_a(L) \).

For both majority voting and the MLE classification strategies, the infidelity may be expressed exactly in terms of the likelihoods

\[
1 - \mathcal{F} = \sum_{a \in \mathcal{A}_0} \lambda_a(L) + \sum_{a' \in \mathcal{A}_L} \lambda_{a'}(0), \tag{12}
\]

where \( \mathcal{A}_0 \) \((\mathcal{A}_L)\) is the set of ancilla readout vectors \( a \) which are classified as initial state \( |0\rangle \) \((|L\rangle)\). Whether a given \( a \) falls in either \( \mathcal{A}_0 \) or \( \mathcal{A}_L \) depends on the classification strategy. By definition, the MLE chooses the sets \( \mathcal{A}_0 \) and \( \mathcal{A}_L \) to be those which minimize the infidelity.

Plots of the infidelity as a function of \( N \) with realistic experimental parameters are shown in Fig. 4 for both majority voting and the MLE. Notably, the minimum infidelity attained by both methods decreases by over an order of magnitude as \( L \) increases.

FIG. 3. Hidden Markov Model for the robust readout scheme. (a) Markov chain and emissions. At each step of the HMM the bosonic system transitions to a new state and releases an emission. Here, \( B_n \) denotes the bosonic mode state after step \( n \), and \( A_n \) denotes the \( n^{th} \) ancilla measurement outcome. (b) Transition and emission probabilities. Transitions and emissions are shown diagrammatically for the case \( L = 2 \), where the matrix elements along the arrows are the associated probabilities. Bolded arrows indicate the intended mappings.

where \( (K_j)_{ij} \) is the transition rate from state \( |j\rangle \) to \( |i\rangle \). For bosonic systems,

\[
(K_j)_{ij} = \begin{cases} 
-j \kappa_j, & i = j \\
-j \kappa_j, & i = j-1 \\
0, & \text{otherwise.}
\end{cases} \tag{8}
\]

This system has the solution \( p(t) = e^{K_j t} p(0) \). The transition probabilities for a level-1 readout taking time \( \tau \) may thus be obtained by explicitly computing the matrix elements of \( e^{K_j \tau} \),

\[
T_{ij}(\tau) = (e^{K_j \tau})_{ji} = \binom{i}{j} (e^{\kappa_j \tau} - 1)^i e^{-\kappa_j \tau}. \tag{9}
\]

To provide intuition as to why increasing the number of levels \( L \) can improve the readout fidelity, we calculate the expected value of the time \( \tau_0 \) which it takes initial state \( |L\rangle \) to decay to \( |0\rangle \),

\[
\langle \tau_0 \rangle = \int_0^\infty d\tau \frac{d}{d\tau} T_{L0}(\tau) = \frac{1}{\kappa_L} \sum_{n=1}^L \frac{1}{n}. \tag{10}
\]
Outperform majority voting as well. It is also clear that the MLE can dramatically improve from 1 to 2. Indeed, the inset shows that increasing $\delta = 2\%$ and $\kappa_L \tau = 1\%$. The minimal infidelities attained by the two methods, however, are not significantly different, meaning that simple majority voting is a near-optimal strategy until decays begin to play a significant role.

To compute the exact infidelity, it is necessary to enumerate all possible combinations of $N$ level-1 readouts and to compute the likelihoods of each, a computation which takes $O(NL^2 \times 2^N)$ operations. To provide a more accessible means of quickly estimating the readout infidelity, and to elucidate its scaling, we derive a simple approximation for the infidelity in the majority voting scheme. The approximation depends on a small number of general experimental parameters: the level-1 readout error probability $\delta$, the decay rate of the bosonic system $\kappa_L$, and the level-1 readout time $\tau$.

There are two dominant processes which are most likely to fool the majority voting. The first is sufficiently quick decay of the initial state $|L\rangle$ to $|0\rangle$, but with no level-1 readout errors occurring. The second is a sufficient number of level-1 readout errors occurring so as to fool the voting, but with no decays occurring. All other processes which fool the voting, such as combinations of decays and level-1 readout errors, have probabilities that are higher order in the parameters $\delta$ or $\kappa_L \tau$. The probabilities of incorrectly identifying initial states may thus be approximated by neglecting the contributions of these higher-order processes,

$$P(0|L) \approx T_{LL} (N \tau) \times \sum_{k=[N/2]}^{L} \binom{N}{k} \delta^k (1 - \delta)^{N-k}$$

$$P(L|0) \approx \sum_{k=[N/2]}^{L} \binom{N}{k} \delta^k (1 - \delta)^{N-k},$$

and expanding to lowest order in $\delta$ and $\kappa_L \tau$ gives

$$1 - F = P(0|L) + P(L|0) \approx 2 \binom{N}{[N/2]} \delta^{[N/2]} + ([N/2]\kappa_L \tau)^L.$$

This approximation is valid when both $N\delta \ll 1$ and $N\kappa_L \tau \ll 1$ so that higher order terms may be neglected. This approximation is plotted along with the exact result in Fig. 4, where the two agree well because the approximation is valid in the regime shown.

Eqn. 14 elucidates the benefit of combining robust encoding with repeated measurement. In two-level systems, such as trapped ions, the fidelity is limited by $\kappa_L \tau$ because $L = 1$ is fixed. On the other hand, in multi-level systems where repetitive QND readouts are not possible, the fidelity is limited by $\delta$ because $N = 1$ is fixed. For bosonic systems in the dispersive coupling regime, however, one has the freedom to increase both $L$ and $N$. Thus, both terms contributing to the infidelity may be suppressed to higher order, and readout is no longer theoretically limited by either individual measurement errors or relaxation. This is the strength of the robust readout scheme.

III. ROBUST READOUT WITH BOTH RELAXATION AND HEATING

We now consider the case where the bosonic mode is subject to heating, defined here as a nonzero excitation rate $\kappa_\uparrow$. Without modification, the readout fidelity of the above scheme would be limited by the probability of the initial state $|0\rangle$ spontaneously exciting to $|1\rangle$, a process which is first order in $\kappa_\uparrow \tau$. In this section, we show how the scheme may be modified so that contributions to the infidelity from heating are also suppressed to higher orders.
The modified readout scheme is shown in Fig. 5, where the excitation rate \(^1\) between the adjacent Fock states \(|n\rangle \) and \(|n + 1\rangle \) is \(n\kappa\). To account for this heating, we define a threshold state \(|m\rangle\) such that the mapping from the bosonic mode to the ancilla is

\[
|n\rangle \rightarrow \begin{cases} |g\rangle, & n \leq m \\ |e\rangle, & n > m. \end{cases} \tag{15}
\]

Hence, the level-1 readouts are described by the measurement operators

\[
\hat{M}_0 = \sum_{k=0}^{m} |k\rangle \langle k|,
\]

\[
\hat{M}_1 = \sum_{k=m+1}^{L} |k\rangle \langle k|. \tag{16}
\]

For \(m > 0\), the contribution to the infidelity from heating of the initial state \(|0\rangle\) will thus be suppressed to higher order in \(\kappa\) because multiple excitations are required for \(|0\rangle\) to heat to a state which is mapped to ancilla state \(|e\rangle\).

![Diagram](image)

FIG. 5. Robust readout scheme for relaxation and heating. Decays and excitations occur between adjacent Fock states with rates proportional to the excitation number. All Fock states \(|n > m\rangle\) are mapped to the excited state of the two-level ancilla.

As in the previous section, this scheme may be quantitatively analyzed with a HMM. The emission probabilities \(E_{ij}\) are similarly defined in terms of the level-1 readout error probability \(\delta\) as

\[
E_{ij} = \begin{cases} 1 - \delta, & \text{if } i \leq m, j = 0 \text{ or } i > m, j = 1, \\ \delta, & \text{otherwise}. \end{cases}
\]

The transition probabilities \(T_{ij}\) are calculated as functions of the decay and excitation rates. The system of differential equations describing the time-evolution of the Fock state populations is

\[
\dot{p}_i(t) = \sum_{j=0}^{L} (K_\uparrow + K_\downarrow)_{ij} p_j(t), \tag{18}
\]

where \(K_\uparrow\) has matrix elements

\[
(K_\uparrow)_{ij} = \begin{cases} -(j+1)\kappa, & i = j < L \\ (j+1)\kappa, & i = j + 1 \\ 0, & \text{otherwise}. \end{cases} \tag{19}
\]

The transition probabilities are then given as a function of the level-1 readout time \(\tau\),

\[
T_{ij}(\tau) = [e^{(K_\uparrow + K_\downarrow)\tau}]_{ji}. \tag{20}
\]

Exact calculations of the infidelity proceed as in the previous section. The infidelity may also be approximated by again considering only the dominant error processes, now including the probability that initial state \(|0\rangle\) heats to \(|m+1\rangle\), with no level-1 readout errors occurring. With this additional term, the level-2 readout error probabilities are approximately given by

\[
\begin{align*}
P(0|L) & \approx T_{LL}(N\tau) \sum_{k=[N/2]}^{L} \binom{N}{k} \delta^k (1-\delta)^{N-k} \\
& \quad + (1-\delta)^N \binom{N}{[N/2]} \kappa \tau \end{align*} \tag{21a}
\]

\[
\begin{align*}
P(L|0) & \approx T_{00}(N\tau) \sum_{k=[N/2]}^{L} \binom{N}{k} \delta^k (1-\delta)^{N-k} \\
& \quad + (1-\delta)^N \binom{N}{[N/2]} \kappa \tau \end{align*} \tag{21b}
\]

To lowest order in \(\delta, \kappa_\downarrow \tau, \) and \(\kappa_\uparrow \tau,\) the infidelity is

\[
1 - \mathcal{F} \approx \binom{L}{m} \left( \binom{N}{2} \kappa_\downarrow \tau \right)^{L-m} \left( \binom{N}{2} \kappa_\uparrow \tau \right)^{m+1} + 2 \left( \binom{N}{[N/2]} \right) \delta^{[N/2]} \tag{22}
\]

It is clear that, within this approximation, all contributions to the infidelity may be suppressed to higher

\(^1\) In order to study the fidelity with a finite HMM, we truncate the Hilbert space to the first \(L+1\) Fock states, taking the heating rate from \(|L\rangle\) to \(|L+1\rangle\) to be 0. The additional levels may be safely neglected for \(\kappa_\uparrow \tau \ll \kappa_\downarrow \tau \ll 1\).
orders in $\kappa_\downarrow \tau$, $\kappa_\uparrow \tau$, and $\delta$, by increasing $L$, $m$, and $N$, respectively.

Plots of the infidelity with both majority voting and the MLE are shown in Fig. 6. Though the heating rates $\kappa_\uparrow$ of physical systems may typically be much smaller than the decay rate $\kappa_\downarrow$ (e.g. [58]), the two are chosen to be comparable in the plot so that the importance of the threshold state is apparent. For the parameters shown in the figure, $m = 0$ is the optimal choice of the threshold for $L \leq 2$, but at $L = 3$ the optimal choice is $m = 1$. In the inset, the minimum majority voting infidelity is plotted as a function of $L$ for both fixed $m = 0$ (red) and the optimal choice of $m$ (black). It is clear that without increasing $m$ the readout infidelity is limited by the first-order heating process, but when $m$ is allowed to increase it is again possible to improve readout fidelity by orders of magnitude. We also note that here again the optimal MLE and majority voting infidelities do not differ significantly.

IV. ROBUST READOUT WITH A MULTI-LEVEL ANCILLA

There exist experimental systems where a bosonic mode can be dispersively coupled to an ancilla with more than two levels. Circuit QED systems provide one example, where higher excited states of a superconducting transmon qubit may be accessed and measured [59, 60]. We now consider a version of the robust readout scheme applicable to such systems and show that the use of a multi-level ancilla can lead to significant improvements in readout fidelity when the MLE is used.

The readout scheme for this case is shown in Fig. 7. As before, nonzero decay and excitation rates are assumed, but in this case the level-1 measurement operators are

$$\{ \hat{M}_k = |k\rangle \langle k|, \text{ for } k = 0, 1, \ldots, L \}. \quad (23)$$

The threshold state $|m\rangle$ is used only to determine which of the $L + 1$ possible ancilla state readouts are counted as votes for initial bosonic state $|0\rangle$ or $|L\rangle$ in the majority voting scheme. It plays no role in the MLE.

(a)

$$\begin{align*}
|L\rangle & \xrightarrow{L\kappa_\downarrow} |L\rangle \\
|L - 1\rangle & \xrightarrow{L\kappa_\uparrow} |L - 1\rangle \\
|m + 1\rangle & \xrightarrow{m} |m + 1\rangle \\
|m\rangle & \xrightarrow{m} |m\rangle \\
|2\rangle & \xrightarrow{2\kappa_\downarrow} |2\rangle \\
|1\rangle & \xrightarrow{2\kappa_\uparrow} |1\rangle \\
|0\rangle & \xrightarrow{\kappa_\downarrow \tau, \kappa_\uparrow \tau} |0\rangle
\end{align*}$$

Votes for $|L\rangle$

Votes for $|0\rangle$

(b)

$$\begin{align*}
|\psi\rangle_B & \xrightarrow{U_{DC}} |\hat{F}_{L+1}^{-}\rangle \\
|0\rangle_A & \xrightarrow{U_{L+1}} |\hat{F}_{L+1}^{-}\rangle
\end{align*}$$

FIG. 6. Infidelity of the robust readout scheme with both relaxation and heating. The infidelity is plotted as a function of the number of measurements $N$ for $L = 1, 2$ and $3$, with the parameter choices $\delta = 2\%$, $\kappa_\downarrow \tau = 1\%$, and $\kappa_\uparrow \tau = 0.5\%$. Inset: the minimum attainable infidelity as a function of $L$. For $m = 0$ (red) the infidelity asymptotes to a finite value, but for optimal $m$ (black) it continues to decrease.

FIG. 7. Robust readout scheme for multi-level ancilla. (a) Schematic description. Both decays and excitations occur between adjacent bosonic mode Fock states. Each Fock state is mapped to a unique ancilla state. In the majority voting all ancilla readouts of $|n > m\rangle$ are counted as votes for $|L\rangle$, while readouts of $|n \leq m\rangle$ are counted as votes for $|0\rangle$. (b) Mapping circuit. Fourier gates on the ancilla can be used in combination with evolution under the dispersive coupling to implement the mapping in a QND way.
states. For a bosonic mode dispersively coupled to an \((L+1)\)-level ancilla, the coupling Hamiltonian is

\[
\hat{H}_{DC}/\hbar = -\sum_{j=0}^{L} \chi |j\rangle \langle j| \hat{a}^\dagger \hat{a}, \tag{24}
\]

where \(|j\rangle\) are the ancilla states. The bosonic mode and ancilla are allowed to evolve under this coupling for a time \(t = 2\pi/(L+1)\chi\), implementing the unitary

\[
\hat{U}_{DC} = e^{i \frac{2\pi}{L+1} (j|\hat{a}^\dagger \hat{a}|j)}, \tag{25}
\]

after which the application of the gate \(\hat{F}_{L+1}^j\) completes the mapping of the bosonic mode’s excitation number onto the ancilla. This mapping is QND because the measurement operators \(M_k\) commute with the dispersive coupling.

The HMM transition probabilities \(T_{ij}\) are the same as in the previous section, but it is necessary to redefine the emission probabilities \(E_{ij}\) to incorporate the \(L+1\) possible ancilla readouts. We define the emission matrix elements

\[
E_{ij} = \begin{cases} (1 - \delta), & \text{for } i = j \\ \delta/L, & \text{otherwise}. \end{cases} \tag{26}
\]

This choice\(^2\) is made so that \(\delta\) remains an easily measurable parameter: given the ability to reliably prepare an initial Fock state, \((1 - \delta)\) is measurable as the probability that the state is correctly read out as the corresponding ancilla state.

As before, the infidelity of the level-2 readout for both the majority voting and MLE may be calculated exactly, or approximately for the majority voting scheme:

\[
1 - \mathcal{F} \approx \begin{pmatrix} L \\ m \end{pmatrix} \left( \begin{pmatrix} N/2 \\ \kappa_+ \tau \end{pmatrix} \right)^{L-m} + \begin{pmatrix} N/2 \\ \kappa_+ \tau \end{pmatrix}^{m+1} \\
+ \begin{pmatrix} N \\ \lfloor N/2 \rfloor \end{pmatrix} \left[ \begin{pmatrix} (m+1)/L \delta \\ \Phi \end{pmatrix} \right] + \begin{pmatrix} (L-m)/L \delta \\ \Phi \end{pmatrix} \right]. \tag{27}
\]

Representative infidelities are plotted in Fig. 8. The most salient feature of the plot is the discrepancy between the minimum infidelities attained by the majority voting and the MLE. Whereas in the previous cases the two were not found to differ significantly, here the MLE is a clearly superior strategy. This discrepancy is due to the fact that the majority voting uses only binary information (votes for \(|0\rangle\) or \(|L\rangle\)) to classify the \(N\) level-1 outcomes. In contrast, the MLE can take any of the \(L+1\) possible ancilla readouts as input and thus extracts more information from each level-1 readout. With this additional information, the MLE is able to more accurately determine the initial state. We further explore an information-theoretic description of the robust readout scheme in the next section.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig8.png}
\caption{Infidelity of the robust readout scheme with multi-level ancilla. The infidelity is plotted with the parameter choices \(\delta = 2\%, \kappa_+ \tau = 1\%\), and \(\kappa_+ \tau = 0.5\%.\) The dashed line is a lower bound on the fidelity determined through information-theoretic considerations (see Sec. V).}
\end{figure}

\section{V. INFORMATION-THEORETIC DESCRIPTION}

In this section, we consider the fidelity of the robust readout scheme from the perspective of classical information theory. The initial state of the bosonic mode constitutes one bit\(^3\) of information, and it is the goal of the robust readout scheme to extract as much of this information as possible. By quantifying the amount of information extracted, it is possible to place a general lower bound on the readout infidelity.

We treat the initial state of the bosonic mode as a classical discrete random variable \(B\) and suppose

\begin{itemize}
\item[\(^2\)] Note that with this definition \(\delta\) is no longer the probability of obtaining a misleading readout. As a result, expressions involving \(\delta\) in this section are not directly comparable to those in previous sections.
\item[\(^3\)] In this section, all logarithms are base 2.
\end{itemize}
that initial states \(|0\rangle\) and \(|L\rangle\) are equally likely,

\[ p_B(b) = \frac{1}{2}, \quad \text{(28)} \]

where \(b \in \{0, L\}\) is a realization of \(B\). Similarly, we treat the series of \(N\) ancilla readouts as a discrete random variable \(A\). The conditional probability distribution of \(A\) given \(B\) is given by the likelihood

\[ p_{A|B}(a|b) = \lambda_a(b) \]

\[ = \sum_{j_1 \ldots j_N} T_{bj_1} E_{j_1a_1} \ldots T_{j_N-1j_N} E_{j_Na_N}, \quad \text{(29)} \]

where \(a\), an \(N\)-vector whose components are the ancilla measurement outcomes, is a realization of \(A\). (For a two-level ancilla, \(a_i \in \{0, 1\}\), while for an \((L+1)\)-level ancilla \(a_i \in \{0, 1, \ldots, L\}\).) The remaining distributions may also be computed in terms of the likelihoods: the joint probability distribution for \(A\) and \(B\),

\[ p_{AB}(a, b) = \frac{1}{2} \lambda_a(b); \quad \text{(30)} \]

the marginal probability distribution for \(A\),

\[ p_A(a) = \frac{\lambda_a(0) + \lambda_a(L)}{2}; \quad \text{(31)} \]

and the conditional probability distribution of \(B\) given \(A\),

\[ p_{B|A}(b|a) = \frac{\lambda_a(b)}{\lambda_a(0) + \lambda_a(L)}. \quad \text{(32)} \]

The bosonic mode’s initial state contains one bit of information, as quantified by the entropy \(H\) of random variable \(B\),

\[ H(B) = -\sum_b p_B(b) \log(p_B(b)) = 1. \quad \text{(33)} \]

The goal of the robust readout scheme is to indirectly extract as much of this information as possible through random variable \(A\). The conditional entropy

\[ H(B|A) = -\sum_{a,b} p_{AB}(a,b) \log(p_{B|A}(b|a)), \quad \text{(34)} \]

quantifies the amount of uncertainty in \(B\) given \(A\), and it follows that the mutual information

\[ I(A; B) = H(B) - H(B|A) \quad \text{(35)} \]

quantifies the amount of information extracted through the robust readout procedure.

These quantities may be used to bound the readout fidelity. Consider a classification process where one attempts to determine \(B\) from \(A\). Let \(\hat{B}(A)\) be the guessed value of \(B\). The probability of an incorrect assignment \(P(\hat{B}(A) \neq B) \equiv p_c\) is related to the conditional entropy through Fano’s inequality,

\[ H(B|A) \leq H_2(p_c) + p_c \log(|B| - 1). \quad \text{(36)} \]

Here, \(B\) is the support of random variable \(B\), and \(H_2\) is the binary entropy,

\[ H_2(p_c) = -p_c \log(p_c) - (1 - p_c) \log(1 - p_c). \quad \text{(37)} \]

Thus, given the relaxation and heating probabilities, \(\kappa_I\tau\) and \(\kappa_F\tau\), and the level-1 readout error probability \(\delta\), the infidelity of the robust readout scheme \((1 - F) = 2p_c\) may be lower-bounded by calculating the conditional entropy, then solving for the value of \(p_c\) which saturates Fano’s bound.

This lower bound is shown in Fig. 8 for the case of \(L = 3\). This bound behaves similarly to the MLE, since the MLE is the optimal classification strategy. Despite the fact that the bound is not saturated, it is clear from the figure that classical information theory provides a reasonable alternative perspective from which the fidelity of the robust readout scheme can be understood.

For completeness, we show why the MLE does not attain the bound. The bound is saturated only if \(H(B|A) = H_2(p_c)\), since \(|B| = 2\). This condition may be rewritten as \(H(E|A) = H(E)\), where \(E\) is the discrete random variable

\[ E = \begin{cases} 1, & \hat{B} \neq B \\ 0, & \hat{B} = B. \end{cases} \quad \text{(38)} \]

Qualitatively, \(H(E|A) = H(E)\) holds when \(A\) does not provide any information about whether a classification error will happen, i.e. when classification errors are equally likely for all realizations of \(A\). This property does not generally hold for the robust readout scheme since typically \(P(0|L) \neq P(L|0)\). This is a consequence of the asymmetry between relaxation and heating rates, which enables one to be more confident in a correct classification for some sequences of ancilla readouts over others.

VI. ALTERNATE ENCODINGS

Given a qubit stored in a bosonic mode as \(| \psi \rangle_B = \alpha |0\rangle_B + \beta |1\rangle_B\), we have thus far only considered readout using the Fock state encoding

\[ |0\rangle_B = |0\rangle \]

\[ |1\rangle_B = |L\rangle. \quad \text{(39)} \]
This choice was made for simplicity—with this encoding the readout fidelity can be computed classically. While this encoding could be useful in applications where high-fidelity readout is prioritized, e.g. for communication qubits in a modular quantum computing architecture [62–64], it may not be ideal for more general applications. Thus, in this section we consider alternate encodings. We develop a set of sufficient encoding criteria for the robust readout procedure to be applicable, show how these criteria are satisfied by cat codes and binomial codes, and approximate the majority voting readout fidelity for both encodings.

### A. Encoding criteria

For a qubit encoded in a lossy bosonic mode as $|\psi\rangle_B = a |0\rangle_B + \beta |1\rangle_B$, we identify three encoding criteria that are sufficient for robust, ancilla-assisted readout in the $\{|0\rangle_B, |1\rangle_B\}$ basis.

**Criterion 1:** Encodings must be robust against excitation loss so that a single loss error cannot destroy all information about the initial state. Explicitly, when subject to $k$ excitation losses, let the logical states $|0\rangle_B$ and $|1\rangle_B$ be respectively mapped to error states $|E^0_k\rangle$ and $|E^1_k\rangle$. The encoding is said to be robust against $d$ excitation losses if

$$\langle E^0_k | E^1_\ell \rangle = 0, \text{ for } k \text{ and } \ell \in \{0, 1, \ldots, d\},$$

where $|E^0_k\rangle$ ($|E^1_\ell\rangle$) denotes $|0\rangle_B$ ($|1\rangle_B$). For example, the Fock state encoding (39) is robust against $d = L - 1$ excitation losses. We note that this criterion is less stringent than the Knill-Laflamme conditions for quantum error correction [65] because we only need to protect a bit of classical information.

**Criterion 2:** The two logical states and their corresponding error states must be distinguishable through an ancilla readout procedure that is QND. For a projective measurement described by $\{\hat{M}_k\}$ that is capable of distinguishing these states, the measurement is QND if

$$\left[\hat{H}(t), \hat{M}_k\right] = 0 \text{ for all } k,$$

where $\hat{H}(t)$ is the Hamiltonian describing the readout procedure. The satisfaction of this criterion enables repeated readouts. As an example, a measurement described by the operators

$$\hat{M}_0 = |0\rangle_B \langle 0|_B + |E^0_\ell\rangle \langle E^0_\ell| + \ldots + |E^0_d\rangle \langle E^0_d|$$

$$\hat{M}_1 = |1\rangle_B \langle 1|_B + |E^1_\ell\rangle \langle E^1_\ell| + \ldots + |E^1_d\rangle \langle E^1_d|,$$

is capable of distinguishing the logical states and their corresponding error states, and it is QND if both $\hat{M}_0$ and $\hat{M}_1$ commute with $\hat{H}(t)$. For the two-level ancilla readout procedure of Sec. II, the measurement operators (3) commute with the dispersive coupling Hamiltonian, thereby satisfying this criterion.

**Criterion 3:** Ancilla errors must not induce damaging changes in the bosonic mode’s state. Let possible ancilla errors be described by a set of jump operators $\{\hat{J}_\ell\}$. For an ancilla error occurring at time $t$ during a level-1 readout, the evolution of the combined system is described by the operator

$$\hat{J}_\ell(t) = T e^{-\frac{i}{\hbar} \hat{J}_\ell^\dagger \hat{H}(t')dt'} \hat{J}_\ell T e^{-\frac{i}{\hbar} \hat{J}_\ell^\dagger \hat{H}(t')dt'},$$

where $T$ denotes time-ordering. We must have

$$\left[\hat{J}_\ell(t), \hat{M}_k\right] = 0, \text{ for all } k \text{ and } \ell,$$

so that ancilla jumps do not affect measurement outcomes by altering the bosonic mode state.

More concretely, for a $d$-level ancilla we consider the possible ancilla errors

$$\hat{J} \in \{|n\} \langle m|, \text{ for } n \neq m \text{ and } n, m \leq d \},$$

corresponding to spontaneous transitions of the ancilla state. In the dispersive coupling regime, such jumps induce dephasing of the bosonic mode that can be modeled as applications of the operator $\hat{J}^\dagger \sim \hat{n}$ and its higher powers [39, 50]. Therefore, we must have $[\hat{n}, \hat{M}_k] = 0$ for this criterion to be satisfied, lest readout fidelity be limited by the probability of spontaneous ancilla transitions.

These three criteria are satisfied by the Fock state encoding (39). We now show explicitly that the criteria are also satisfied by cat codes and binomial codes, and we approximate the fidelity of the robust readout scheme for both types of codes.

### B. Cat codes

Cat codes [37, 61, 66, 67] are quantum error correcting codes designed to protect against excitation loss. Quantum error correction with cat codes has recently reached the break-even point where the lifetime of encoded qubits exceeds the lifetimes of all constituent components [10]. The codewords are formed from equal superpositions of coherent states. Let the state $|C^\alpha_n\rangle$ be defined as a superposition of $2L$ coherent states evenly distributed around a circle in the bosonic mode’s phase space

$$|C^\alpha_n\rangle = \frac{1}{2L\sqrt{N^\alpha}} \sum_{k=0}^{2L-1} e^{-ikn\pi/L} |e^{ik\pi/L}\rangle,$$
where \( N_n^\alpha \) is a normalization factor \([61]\). These superposition states may be expressed in terms of Fock states as

\[
|C^\alpha_n\rangle = \frac{1}{\sqrt{N_n^\alpha}} \sum_{m=0}^\infty e^{-|\alpha|^2/2} \alpha^{n+2mL} |n+2mL\rangle_F
\]

(47)

where the subscript \( F \) is used in this section to distinguish Fock states from coherent states. It is important to note that \( |C^\alpha_n\rangle \) is a superposition of Fock states which all have the same excitation number \( n \) modulo 2\( L \). We define the logical states

\[
|0\rangle_B = |C^L_0\rangle \\
|1\rangle_B = |C^2L_0\rangle.
\]

(48)

**Criterion 1.** After \( k \) excitation loss events, the state \( |C^\alpha_{n}\rangle \) is mapped to \( |C^\alpha_{n-k}\rangle \). The cat codes are robust against \( L - 1 \) excitation loss events since

\[
\langle C^L_{n-k} | C^{2L-\ell}_n \rangle = 0, \text{ for } k, \ell \leq L - 1.
\]

(49)

**Criterion 2.** The cat code logical states and their corresponding error states can be distinguished by measurement of the excitation number modulo 2\( L \). This measurement can be described by the set of measurement operators \( \{\hat{M}_k, k = 0, \ldots, 2L - 1\} \), where

\[
\hat{M}_k = \sum_{m=0}^\infty |k+2Lm\rangle \langle k+2Lm|.
\]

(50)

This measurement may be implemented using the dispersive coupling \( \hat{H}_{DC} \) with a procedure similar to the one shown in Fig. 7(b). For a bosonic mode dispersively coupled to a 2\( L \)-level ancilla, it is possible to realize the unitary

\[
\hat{U} = F_{2L} e^{i2\pi |j\rangle \langle j|a^d a} F_{2L}^†,
\]

(51)

which maps the bosonic mode’s excitation number modulo 2\( L \) onto the ancilla. This measurement process is QND because \( [\hat{H}_{DC}, \hat{M}_k] = 0 \) for all \( k \).

**Criterion 3.** Spontaneous ancilla transitions during the readout process do not induce damaging changes in the bosonic mode’s state because the measurement operators \( \hat{M}_k \) commute with dephasing errors \( \hat{n} \) for all \( k \).

**Fidelity.** To approximate the fidelity of the majority voting scheme we consider the two processes most likely to fool the voting: (1) sufficient level-1 readout errors with no excitation loss events, and (2) \( L \) excitation loss events occurring sufficiently quickly with no level-1 readout errors. The probability of process (1) can be computed in terms of \( \delta \), the probability of obtaining a misleading level-1 readout, as in the previous sections. To compute the probability of process (2), we first note that the Kraus operator-representation for the lossy bosonic channel \([68]\) is

\[
\mathcal{L}(\hat{\rho}) = \sum_{k=0}^\infty \hat{A}_k \hat{\rho} \hat{A}_k^†,
\]

(52)

where

\[
\hat{A}_k = \sqrt{\frac{(1-e^{-\kappa \tau} k!)}{k!}} e^{-\kappa \tau} a_k
\]

(53)

is the Kraus operator corresponding to \( k \) excitation losses. The probability of process (2) is the probability of initial state \( |C^\alpha_n\rangle \) suffering \( L \) excitation loss events in a time \([N/2]\tau\), which is approximately given by

\[
\langle \hat{A}_L^† \hat{A}_L \rangle \approx \frac{\delta^{[N/2]}}{L!} \langle \hat{a}^d a L \rangle L \cdot (|\alpha|^2[N/2]\kappa \tau)^L.
\]

(54)

To lowest order in \( \delta \) and \( \kappa \tau \), the cat code readout fidelity \( F_{\text{cat}} \) is thus given by

\[
F_{\text{cat}} \approx 1 - 2 \left( \frac{N}{[N/2]} \right) \delta^{[N/2]} - \frac{2}{L!} (|\alpha|^2[N/2]\kappa \tau)^L.
\]

(55)

Within this approximation it is clear that both error terms may be suppressed to higher order. The contribution from individual measurement infidelity can be suppressed by increasing \( N \), and the contribution from excitation loss can be suppressed by increasing the number of coherent states comprising the cat state—analogous to increasing the excitation number used in the Fock state encoding.

**C. Binomial codes**

Binomial codes \([39]\) are a new class of quantum error correcting codes that can protect against excitation loss and gain errors as well as dephasing errors. The codewords are formed from superpositions of Fock states weighted with binomial coefficients

\[
|0\rangle_B = \frac{1}{\sqrt{2^{M-1}}} \sum_{p \text{ even}}^{[0,M]} \sqrt{M \choose p} |pL\rangle,
\]

\[
|1\rangle_B = \frac{1}{\sqrt{2^{M-1}}} \sum_{p \text{ odd}}^{[0,M]} \sqrt{M \choose p} |pL\rangle,
\]

(56)

where \( M \) and \( L \) are positive integers, and the range of the index \( p \) is from 0 to \( M \).
Criterion 1. The error state \( |E_0^k\rangle\) is a superposition of Fock states with excitation number \( L - k \) mod 2\( L \), while error state \( |E_1^\ell\rangle\) is a superposition with excitation number \( 2L - \ell \) mod 2\( L \). Therefore, the binomial codes are robust against \( L - 1 \) excitation loss events since \( \langle E_0^k|E_1^\ell\rangle = 0 \) for \( k \) and \( \ell \) between 0 and \( d \).

Criterion 2. The binomial code logical states and corresponding error states can be distinguished by measuring the excitation number modulo 2. This measurement (50) is the same as that considered for cat codes, and it is QND by the same argument.

Criterion 3. Spontaneous ancilla transitions during the readout process do not induce damaging changes in the bosonic mode’s state by the same argument as for cat codes.

Fidelity. We approximate the fidelity of a major- ity voting scheme by considering the two processes most likely to fool the voting. The argument here proceeds analogously to the one given for cat codes, except that the probability of process (2) is different for binomial codes. The probability that one of the initial states (56) suffers \( L \) excitation loss events in a time \( [N/2]\tau \) is approximately given by

\[
\langle \hat{A}_L^\dagger \hat{A}_L \rangle \approx \frac{([N/2]\kappa_\delta^1 \tau)^L}{L!} \langle \hat{a}^L \hat{a}^L \rangle \\
= \frac{1}{L!} \left( \frac{LM}{2} \frac{[N/2]\kappa_\delta^1 \tau}{2} \right)^L.
\]

To lowest order in \( \delta \) and \( \kappa_\delta^1 \tau \), the binomial code read- out fidelity \( F_{\text{bin}} \) is then given by

\[
F_{\text{bin}} \approx 1 - 2 \left( \frac{N}{[N/2]} \right) \delta^{[N/2]} \frac{2}{L!} \left( \frac{LM}{2} \frac{[N/2]\kappa_\delta^1 \tau}{2} \right)^L.
\]

As with the cat codes, it is clear that both error terms may be suppressed to higher orders.

VIII. CONCLUSIONS

We have shown how the combination of robust en- coding and repeated QND measurements constitutes a powerful means of improving qubit readout fidelity. Robust encodings allow one to suppress contributions to the infidelity from relaxation, and repeated QND measurements allow one to suppress contributions from individual measurement infidelity. For bosonic systems in the dispersive coupling regime, these techniques may be simultaneously applied. Dispersive couplings have been experimentally demonstrated in circuit QED [42, 44, 45] and optomechanical systems [46–48], and for these systems the robust readout scheme could be readily applied, potentially yielding orders of magnitude improvement in readout fidelity. While in this work we have studied the fidelity of the scheme for a single Fock state encoding, we also describe how the scheme could be applied to either cat or binomial codes. Ultra-high-fidelity logical state readout would be of great practical use in a number of applications where measurement fidelity is prioritized, including gate teleportation, entanglement purification, and modular quantum computation.

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