Radio-Frequency Single-Electron Transistor as Readout Device for Qubits: Charge Sensitivity and Backaction

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We study the radio-frequency single-electron transistor (rf-SET) as a readout device for charge qubits. We measure the charge sensitivity of an rf-SET to be $6.3\mu e/\sqrt{\mathrm{Hz}}$ and evaluate the backaction of the rf-SET on a single Cooper-pair box. This allows us to compare the needed measurement time with the mixing time of the qubit imposed by the measurement. We find that the mixing time can be substantially longer than the measurement time, which would allow readout of the state of the qubit in a single shot measurement.

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A large number of physical systems have been suggested as basic elements for qubits in quantum computers. In terms of scaling the system to a large number of qubits, the solid state systems have clear advantages [1–5]. The qubit system which we will discuss in this paper is the so-called single Cooper-pair box (SCB) [6] (see Fig. 1a), which operates based on the Coulomb blockade [7,8] and has been suggested as one of the qubit candidates [3].

The Cooper-pair box is a small superconducting island connected via a Josephson junction to a reservoir. It is described by its Josephson coupling energy E_J, and by its charging energy $E_C = e^2/2C_{qb}$, where C_{qb} is the total capacitance of the box and e is the electron charge. In the SCB it is possible to create superpositions [6] of charge states involving a discrete number of excess Cooper pairs in the box, which we denote $|0\rangle$, $|1\rangle$, $|2\rangle$, etc. Because of the Josephson coupling energy, the states form Bloch bands as a function of an external voltage V_{qb} (see Fig. 1). If E_J is much smaller than E_C, only neighboring states are relevant and with an appropriate V_{qb} two states can be selected, for example, $|0\rangle$ and $|1\rangle$. The superconducting energy gap Δ of the box has to be larger than the charging energy in order to avoid quasiparticle states, and finally temperature has to be small compared to all these energy scales. Thus the inequality $\Delta > E_C \gg E_J \gg k_BT$ has to be satisfied in order for the system to act as a well-defined two level system.

Quantum coherence in a SCB was demonstrated in an experiment by Nakamura *et al.* [9]. They showed how the states could be manipulated using very fast voltage pulses, and they observed coherent oscillations between the two states by varying the pulse duration.

An important part of any qubit concept is the readout system, which has to be sufficiently sensitive and fast. The measurement decoheres the qubit, but the backaction of the measurement must be sufficiently weak not to induce mixing between the states. A single shot measurement would be a clear advantage when implementing error correcting algorithms [10,11] in a quantum computer. The obvious readout device for the SCB qubit would be a single-electron transistor (SET) [12–14] which is able to detect subelectron charge variations. With the invention of the radio-frequency single-electron transistor (rf-SET) [15] the SET transistor can also be made very fast.

The question we address in this paper is whether it is possible to read out the state of a single Cooper-pair box with an rf-SET in a single shot measurement. The experiment we discuss is similar to the one by Nakamura *et al.,* with the important difference that with the rf-SET we can turn the measurement on and off. In our analysis we present our results in two steps. First we present measurements of an rf-SET which shows very high sensitivity. From this data we can evaluate the measurement time t_m, needed to resolve the two states of the qubit. Second, using the experimentally measured quantities for the rf-SET, and assuming that the rf-SET is coupled to an aluminum SCB qubit via a coupling capacitance C_c, we calculate the backaction which the measurement would have on the qubit, and thus evaluate the mixing time, t_{mix}, in the qubit. This allows us to compare the needed measurement time with the mixing time.

Here we focus on the single Cooper-pair box, but it is important to note that fast detection of subelectron charge variations may be relevant also for other qubit schemes [16], and the results presented here are thus of more general interest.

The measured SET transistor was fabricated by electron-beam lithography and standard two-angle evaporation of aluminum, with oxidation after the first layer to create tunnel junctions. The total resistance of the SET was 44.1 kΩ, and the sum capacitance was $C_{\mathrm{SET}} = 370$ aF, corresponding to $E_{\mathrm{SET}} = e^2/(2C_{\mathrm{SET}}) = 2.5$ K. The current-voltage (IV) characteristic of the SET could be modulated with a gate voltage as shown in Fig. 2. The gate voltage period was $\Delta V_g = 10$ mV corresponding to a gate capacitance $C_g = 16$ aF. The sample was mounted at the mixing chamber of a dilution refrigerator which had a base temperature of about 20 mK.

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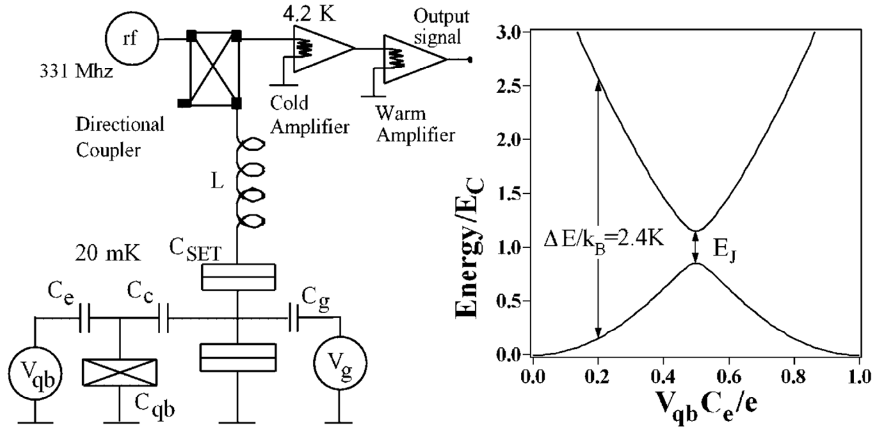

FIG. 1. (a) The measurement circuit of the radio-frequency single-electron transistor, and how it could be used to read out a qubit consisting of a single Cooper-pair box. (b) Energy levels for the single Cooper-pair box as a function of the external voltage, V_{qb}.

The drain of the SET was connected to a chip inductor, while the source was grounded. The value of the inductor (620 nH) was chosen so that it formed a resonant circuit with the capacitance C_p of the substrate pad at a frequency $f_0 \approx 331$ MHz. Figure 1a shows the schematic of the measurement setup. The carrier rf signal was supplied by a network analyzer and launched to the tank circuit via a number of attenuators and a directional coupler.

To evaluate the tank circuit parameters, we applied a large dc current (1 μA) through the SET and detected the resulting shot noise with a spectrum analyzer. Good agreement between the calculated and the measured power noise can be seen in the upper left inset of Fig. 2. We estimate the quality factor of the tank circuit to be $Q = 18$ [17], and the bandwidth of the system to be 9 MHz.

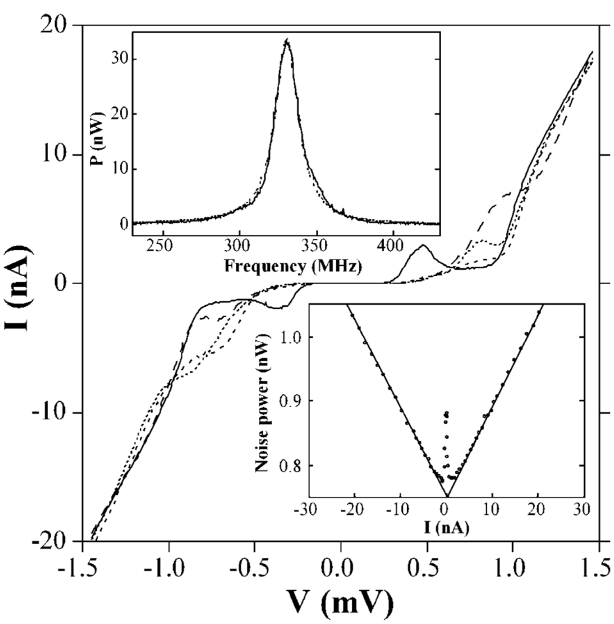

FIG. 2. IV characteristics of the single-electron transistor in the superconducting state, the different curves are for different gate voltages. The upper left inset shows measured and calculated shot noise from the SET, for a current of 1 μA. The lower right inset shows the output noise power of the system as a function of current through the SET; the peak around zero bias is due to the amplifier, which experiences a high source impedance.

A cold amplifier with 24 dB gain was situated in the helium bath. Two more amplifiers with a total gain of 53 dB were placed at room temperature. The reflected and amplified signals could either be measured with a spectrum analyzer, or could be detected with a diode detector and recorded using a sampling oscilloscope.

To evaluate the noise temperature of the whole system, we measured the noise power as a function of the current through the SET (see Fig. 2). From the slope of the linear parts, originating from the shot noise in the SET, we extract the total gain of the system, which was found to be 77.2 dB. From the crossing point of the linear asymptotes, we extract the noise temperature of the amplifier system, which was 10.3 K. Note that when the SET is not loading the tank circuit, i.e., when the SET is in the Coulomb blockade state, the amplifier experiences a different source impedance, and thus the noise contribution from the amplifier is higher, causing the peak at zero bias in the lower right inset of Fig. 2. In this case we get a noise temperature of 12.0 K. The warm amplifiers contribute to the noise temperature with $\sim$1 K.

For a given carrier signal at the resonance frequency f_0, the reflected power from the SET was amplified and detected as a function of V_g. The reflected signal showed the typical periodic response as a function of the gate voltage corresponding to the addition of individual electrons to the island [15].

Figure 3 shows the two sidebands of the amplitude-modulated carrier for a gate signal with an amplitude of $0.1e_{\rm rms}$ and at 2.1 MHz. The noise floor within ± 1 MHz of the main peak is high due to the relatively high phase noise of the carrier source.

Measuring the signal-to-noise ratio (SNR) of the side peak and changing the rf amplitude, we can optimize the rf amplitude. The right inset of Fig. 3 shows the response to a $0.0085e_{\rm rms}$ gate signal at 2 MHz as a function of rf amplitude. The best SNR of 21.7 dB, obtained with a

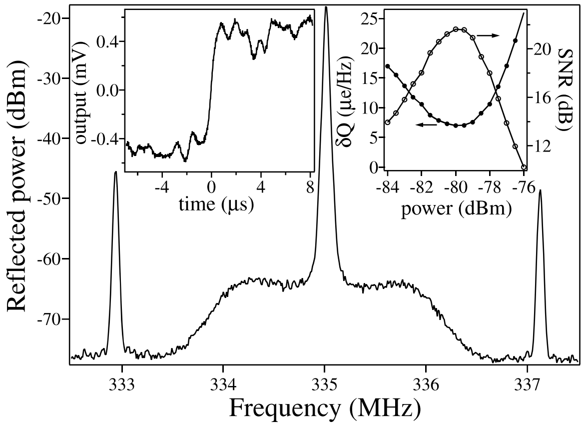

FIG. 3. The reflected power as a function of frequency. The carrier is amplitude modulated by the SET, generating two sidebands, when a signal of $0.1e_{\rm rms}$ and 2.1 MHz is applied to the gate. Note the very large signal-to-noise ratio, and that the phase noise of the carrier source dominates within ± 1 MHz around the main peak. The right inset shows the charge sensitivity as a function of the carrier amplitude with a gate signal corresponding to $0.0085e_{\rm rms}$. The left inset shows the rf-SET response to a $0.2e$ step function at the gate, with no averaging.

resolution bandwidth of 10 kHz, corresponds to a charge sensitivity of $\delta q = 6.9\mu e/\sqrt{\rm Hz}$. For slightly different parameters of carrier frequency and dc-gate voltage we obtained $\delta q = 6.3\mu e/\sqrt{\rm Hz}$, which corresponds to an uncoupled energy sensitivity $\delta\epsilon = (\delta q)^2/(2C_{\rm SET}) = 13\hbar$. The maximum SNR for an rf-SET operated in the superconducting state should occur at a power where the amplitude of the rf signal corresponds to the gap voltage. We estimate that the optimum should occur at $P_{\rm opt} \approx -78$ dBm which compares well with our experimental data.

The ultimate sensitivity should be limited by shot noise, and it has been estimated for a dc operated SET in the normal state to $\delta\epsilon \approx 0.7\hbar$ [18]. For an rf-SET the sensitivity is expected to be a factor 1.4 worse [19]. Assuming that our superconducting rf-SET would have a sensitivity similar to a normal state device, we could thus hope to reach $\delta\epsilon \approx \hbar$.

From our noise measurements in the normal state (see the insets of Fig. 2) we can estimate the shot noise and the amplifier contributions. At the rms current 6.7 nA, which is where we find the optimum sensitivity, we find a shot noise addition of approximately 93 pW (referred to the output of the system, and using a resolution bandwidth of 100 kHz). However, because of the correlated tunneling in the superconducting state, we expect twice as much noise compared to the normal state that is 186 pW. This should now be compared to the noise from the amplifier, which varies from 750 to 880 pW depending on the state of the SET. Summarizing, we estimate that only 20%–25% of the noise comes from the shot noise and that the rest comes from the amplifier. This indicates that the shot noise contributes $\sim 3\hbar$, and that the amplifier contributes $\sim 10\hbar$ to our $\delta\epsilon$.

A factor which further limits the sensitivity of our system is the asymmetric rf bias of the SET. When the rf signal is positive, an extra gate charge of $C_g V_g$ is added, and for negative rf bias the same gate charge is subtracted. This leads to an effective gate charge difference between the positive and the negative rf bias of the order of $0.2e$ (using $V = 1$ mV), which leads to a reduction in sensitivity.

The left inset of Fig. 3 shows the time domain response of the rf-SET to a step signal corresponding to $0.2e$ at the gate. The bandwidth was limited to 1 MHz and no averaging was done. The noise level is about $0.007e_{\rm rms}$. This very high charge sensitivity and fast response demonstrate the potential of the rf-SET as a readout device of the quantum state in the SCB.

For a given sensitivity and measurement time t_m, the uncertainty in charge is given by $\Delta q = \delta q/\sqrt{t_m}$. To separate the two qubit states in a real measurement, we need the two intervals $0 \pm \Delta q$ and $2e \pm \Delta q$ not to overlap. We define the coupling $\kappa = C_c/C_{qb}$ and we can thus write the needed measuring time t_m as

$$2e\,\frac{C_c}{C_{qb}} = \frac{2\delta q}{\sqrt{t_m}} \Rightarrow t_m = \left(\frac{\delta q}{\kappa e}\right)^2. \qquad (1)$$

During the measurement, the measurement itself and the presence of the rf-SET can cause transitions between the two states. There are a number of different processes that contribute to this mixing [20]. Here we evaluate the effect of the two processes which we consider to be the most important, namely, the shot noise in the SET and the quantum fluctuations of the qubit's environment. Both of these processes can be described in terms of voltage fluctuations of the SET island. The rate $\Gamma_1 = 1/t_{\rm mix}$ for the transitions is proportional to the spectral density of the voltage noise on the SET island $S_V(\omega)$ at the frequency corresponding to the qubit transition $\Delta E/\hbar$ (see Fig. 1b), and may be evaluated using standard methods [14,18]

$$\Gamma_1 = \frac{1}{t_{\rm mix}} = \frac{e^2}{\hbar^2}\,\kappa^2\,\frac{E_J^2}{\Delta E^2}\,S_V(\Delta E/\hbar)\,. \qquad (2)$$

Equation (2) is valid in the limit used below, $\Delta E \gg E_J$, i.e., away from the degeneracy point of the qubit. Note that the rf carrier has a frequency which is much smaller than the transition frequency and will thus not contribute to the transitions.

The shot noise in the SET may, for low transition frequencies, $\Delta E/\hbar \ll I/e \approx 40$ GHz, be evaluated within "orthodox" SET theory [21]

$$S_V^o(\omega, \omega_I) = 4\,\frac{E_{\rm SET}^2}{e^2}\,\frac{4\omega_I}{\omega^2 + 16\omega_I^2}\,. \qquad (3)$$

Here $\omega_I = I/e$ is the tunneling rate through the SET. We have assumed a symmetric SET and that only single particle tunneling events are relevant.

In addition to the above fluctuations, the qubit couples to electromagnetic modes in the rf-SET. For high frequencies, $\Delta E \gg E_{\rm SET}$, this relaxation process dominates, as can be seen in the inset of Fig. 4. At these frequencies

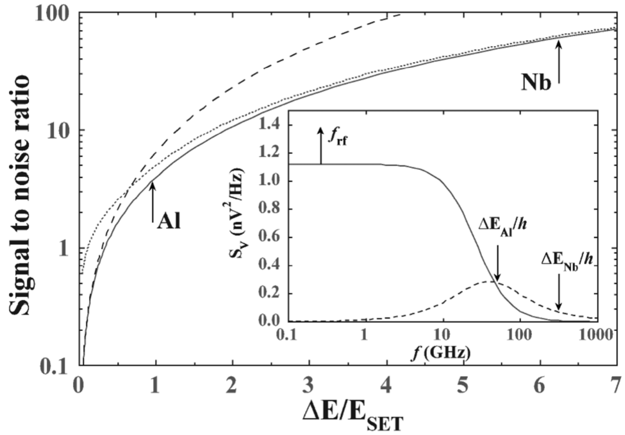

FIG. 4. The signal-to-noise ratio of a single shot measurement as a function of transition frequency. The dashed line shows the SNR due to the shot noise. The dotted line shows SNR due to the quantum fluctuations of the external impedance seen by the qubit. The continuous line shows the result when both rates have been added. The inset shows the power spectral density of the voltage fluctuations of the middle island, $S_V(\omega, I)$ of the rf-SET as a function of frequency. We show the case for $I = 6.7$ $\mathrm{nA_{rms}}$, which gave optimum sensitivity. The solid line shows the spectral density due to shot noise, and the dashed line shows the spectral density due to quantum fluctuations of the environment.

the impedance to ground seen from the SET island may be modeled as two tunnel junctions in parallel, giving

$$S_V^e(\omega) = \hbar\omega \frac{R_t}{1 + (\frac{\hbar\omega}{E_{\mathrm{SET}}} \frac{\pi R_t}{2R_K})^2}, \tag{4}$$

where R_t is the normal resistance of the individual junctions, and $R_K = h/e^2$. The spectral densities are depicted in the inset of Fig. 4.

Adding the rates for these two processes (see Fig. 4) we get an estimate of the mixing time due to the presence of the rf-SET. If the ratio between the mixing time and the measuring time is much larger than unity it should be possible to read out the qubit in a single measurement. Note that this ratio is independent of the coupling capacitance C_c.

For realistic qubit parameters of $E_C/k_B = 1$ K, $E_J/E_C = 0.1$, assuming readout with the qubit biased at $\Delta E/E_C = 2.4$, and using the experimental IV characteristics of the rf-SET with sensitivity $\delta q = 6.3\mu e/\sqrt{\mathrm{Hz}}$, we get SNR $= \sqrt{t_{\mathrm{mix}}/t_m} \approx 4$, thus allowing a single shot measurement.

In principle one should evaluate the noise spectral density of the SET island in a fully quantum mechanical calculation, along the lines of, e.g., Refs. [22] and [23]. Equations (3) and (4) would then be the low- and high-frequency limits of this expression.

Making the qubit from niobium instead of aluminum increases the range of values available for E_J and E_C. Using niobium we may increase E_C by a factor of ~6.5, while keeping E_J constant. This would improve the SNR of a single shot readout by approximately a factor of 15.

In conclusion we have shown that an rf-SET can achieve a charge sensitivity of $6.3\mu e/\sqrt{\mathrm{Hz}}$. We find that the sensitivity is limited by the cold amplifier and that the rf-SET potentially can approach the quantum limit. We show that the ratio between the mixing time and the measurement time can be larger than unity, so that a SCB qubit can be read out with an rf-SET in a single shot measurement. Furthermore we show that SNR depends strongly on the superconducting energy gap of the qubit. Using niobium instead of aluminum in the qubit can improve SNR by more than an order of magnitude.

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