

Quantum Computing with an Electron Spin Ensemble

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We propose to encode a register of quantum bits in different collective electron spin wave excitations in a solid medium. Coupling to spins is enabled by locating them in the vicinity of a superconducting transmission line cavity, and making use of their strong collective coupling to the quantized radiation field. The transformation between different spin waves is achieved by applying gradient magnetic fields across the sample, while a Cooper pair box, resonant with the cavity field, may be used to carry out one- and two-qubit gate operations.

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The construction of a large quantum computer is a challenge for current research. The overarching problem is to develop physical systems which can reliably store thousands of qubits and which allow addressability of individual bits and pairs of bits in gate operations. Proposals in which single trapped ions or atoms encode qubits in their internal state have successfully demonstrated the building blocks for few-bit devices, while scaling of these systems to larger register sizes is believed to require interconnects, e.g., with optical transmission. A novel collective encoding scheme for qubits proposes to use many identical quantum systems to encode each qubit, either in the collective population of different internal states [1–3] or in different spatial modes of excitation of the entire system [4,5].

In this Letter we propose a hybrid approach to quantum computing making use of an ensemble of 10^{10} – 10^{12} electron spins coupled to a superconducting transmission line cavity. We will describe how a large number of spatial modes can be addressed in the spin ensemble, and how a transmon Cooper pair box (CPB) [6] integrated in the cavity can provide one- and two-bit gates for quantum computing in the spin ensemble [7,8]. Our scheme enables materials for which large spin coherence times have been demonstrated in ensemble measurements to be incorporated into a solid-state device. In this way, without requiring single spin measurement or strong coupling to a cavity, full use can be made of the sophisticated techniques which are now well established for control of large numbers of spins.

The proposed physical setup, as illustrated in Fig. 1, consists of a superconducting transmission line cavity coupled to a large number of solid-state electron spins doped into or deposited on the surface of the substrate. Two interesting choices for the electron spins would be *P*-doped Si and endohedral fullerene molecules, e.g.,

$N@C_{60}$, which would offer spin coherence times up to tens of milliseconds [9–11]. Hahn echo techniques may be applied to counter inhomogeneous broadening mechanisms, and the coherence time scale may even be further extended by transferring the electron spin state to nuclear spin degrees of freedom where coherence times exceeding seconds have been demonstrated [12]. The spins are biased with a homogeneous magnetic field B in the plane of the cavity, causing Larmor precession at an angular frequency of $\omega_s = m_0 B / \hbar$, where m_0 is the magnetic dipole moment of the spins. With a cavity resonance frequency of $\omega_c \approx 2\pi \times 5$ GHz, a bias field of $B = 180$ mT is required to bring the spin precession into resonance. Even in the presence of the bias field, cavity linewidths as low as $\kappa \approx 2\pi \times 250$ kHz are possible [13].

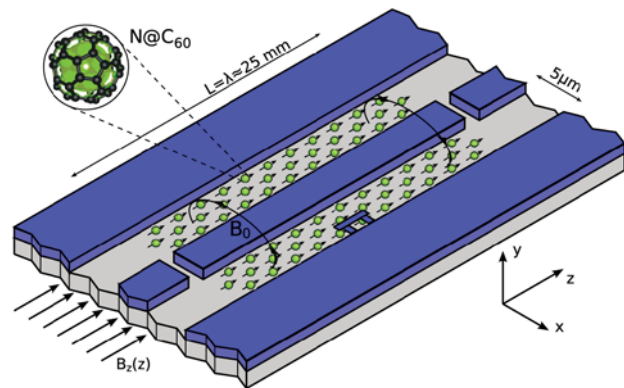


FIG. 1 (color online). Physical setup, consisting of a superconducting transmission line cavity coupled to an ensemble of electron spins and a transmon Cooper pair box [6]. The cavity dimensions allow on the order of $N = 10^{11}$ electron spins to be coupled to the cavity mode with an average coupling strength of $\bar{g} \approx 2\pi \times 20$ Hz. An external magnetic field composed of a homogeneous bias field $B\hat{z}$ and a switchable linear gradient $(z\hat{z} - y\hat{y})\Delta B/L$ is applied to the system.

The q th spin located at \mathbf{r}_q will couple to the cavity annihilation and creation operators, \hat{a} and \hat{a}^\dagger , with a strength $g_q \equiv m_0 B_0(\mathbf{r}_q)/2\hbar$ where $B_0(\mathbf{r})$ is the zero-point magnetic field of the cavity mode. We will use $\bar{g} \equiv \sqrt{\sum_q |g_q|^2}/N$ to denote the average coupling strength.

We now consider the collective coupling between the spin ensemble and the cavity. Assuming the spin-cavity detuning $\Delta_s \equiv \omega_s - \omega_c$ to be small compared to $\omega_{s,c}$, we can apply the rotating wave approximation and describe the coupling by the interaction Hamiltonian

$$\begin{aligned} \hat{H}_{sc} &= \sum_{q=1}^N \hbar g_q (\hat{a}^\dagger \hat{\sigma}_-^{(q)} + \hat{a} \hat{\sigma}_+^{(q)}) - \hbar \Delta_s \hat{a}^\dagger \hat{a} \\ &= \hbar \sqrt{N} \bar{g} (\hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger) - \hbar \Delta_s \hat{a}^\dagger \hat{a}, \end{aligned} \quad (1)$$

where $\hat{\sigma}_-^{(q)}$ ($\hat{\sigma}_+^{(q)}$) is the Pauli spin lowering (raising) operator of the q th spin, and we introduce the collective spin lowering operator $\hat{b} \equiv \frac{1}{\sqrt{N}} \sum_{q=1}^N \hat{\sigma}_-^{(q)} g_q / \bar{g}$ and its Hermitian adjoint \hat{b}^\dagger in the second line. Assuming that the sample is strongly polarized, the collective lowering and raising operators obey the commutator relation $[\hat{b}, \hat{b}^\dagger] = 1$, so that the collective excitation behaves like a harmonic oscillator degree of freedom. The lowest two spin oscillator states are the state with all spins pointing down, $|\text{vac}\rangle \equiv |0\rangle^{\otimes N}$, and the state with a single collective excitation

$$|\psi_1(0)\rangle \equiv \hat{b}^\dagger |\text{vac}\rangle = \frac{1}{\sqrt{N}} \sum_q \frac{g_q}{\bar{g}} |0_1 \dots 1_q \dots 0_N\rangle. \quad (2)$$

According to the second line of Eq. (1) the cavity field and the collective spin oscillator behave as two coupled oscillators with a coupling strength that is enhanced with a factor of \sqrt{N} relative to the single spin coupling strength, so that any states of the two systems are interchangeably mapped between them with the collective Rabi frequency $\sqrt{N} \bar{g}$. For the parameters given in Fig. 1, the effective coupling $\sqrt{N} \bar{g}$ is $\sim 2\pi \times 6$ MHz, and, thus, exceeds by orders of magnitude the decay rates of collective spin excitations (governed by the single spin decay rate [14]) as well as the cavity decay rate κ .

The long coherence lifetimes in trapped atomic and molecular systems have inspired recent proposals to transfer and store the cavity field excitation in a single rotationally excited polar molecule [15], and, to benefit from the collectively enhanced coupling, in collectively excited states of many molecules [16]. In these proposals, molecules would have to be cooled and trapped in the close vicinity of the cavity transmission line, while the electron spins in the present proposal are being held in a host material—at the price of shorter achievable coherence times. In a physical setup similar to ours [17], the energy splitting of the two lower eigenstates of a coupled CPB-cavity system differs from the excitation energies to higher

excited states, and thus they form an effectively closed two-level system which may be resonantly coupled to a collective spin oscillator.

Our objective is to store qubits for quantum computing, and, in particular, to store many qubits in the same medium. To this end, we consider the application of a magnetic field gradient to the sample for a duration τ as described in Fig. 1. The field provides a linearly varying Zeeman energy shift across the sample so that, after a certain interaction time, a linearly varying spatial phase $\exp(ikz)$, with $k = -m_0 \Delta B \tau / L \hbar$, is encoded in the excited state component of the individual spins at position z along the sample, and the collective state $|\psi_1(0)\rangle$ is transferred to

$$|\psi_1(k)\rangle \equiv \frac{1}{\sqrt{N}} \sum_q \frac{g_q}{\bar{g}} e^{ikz_q} |0_1 \dots 1_q \dots 0_N\rangle. \quad (3)$$

Different positive and negative values of the collective spin wave number k can be chosen, and we will in the following refer to the magnetic gradient pulse, accomplishing a specific value of the phase gradient, as a (k) pulse applied to the system. The key idea of the holographic quantum register is that the gradient pulses let us successively access a number of collective excitation modes of the same spin ensemble, such that the read in of each new qubit does not disturb the previously stored qubits, because only the $k = 0$ spin mode of the new qubit interacts with the cavity field. These modes are one dimensional k -space voxels as used in magnetic resonance imaging [18]; similar orthogonal collective atomic excitation modes are being studied for storage of multiple modes of light [5,19–22].

To identify modes which are truly independent, and which can hence be used for storage of different qubits, we introduce the creation operator for the i th spin wave mode as

$$\hat{b}^\dagger(k_i) \equiv \frac{1}{\sqrt{N}} \sum_{q=1}^N \frac{g_q}{\bar{g}} e^{ik_i z_q} \hat{\sigma}_+^{(q)}, \quad (4)$$

and consider the commutator $[\hat{b}(k_i), \hat{b}^\dagger(k_j)]$ in the fully polarized limit. If the spins are arranged on a Bravais lattice and have equal coupling strengths, the commutator vanishes for any pair of members of the reciprocal lattice, so that the corresponding modes are perfectly independent [23]. For a general geometry, we find that $[\hat{b}(k_i), \hat{b}^\dagger(k_j)] = M(k_j - k_i)$, where we have introduced the mode overlap $M(\Delta k) \equiv \sum_q e^{i\Delta k z_q} \frac{|g_q|^2}{N \bar{g}^2}$. For the uniformly doped $L = \lambda$ transmission line cavity illustrated in Fig. 1, $M[\Delta w(2\pi/L)] = \text{sinc}(\pi \Delta w) / [1 - (\Delta w/2)^2]$ in the continuous ($N \rightarrow \infty$) limit, so that in this case, the mode overlap vanishes when the difference in winding number $\Delta w \equiv \Delta k L / 2\pi$ is equal to any integer except 0 or ± 2 . It follows that choosing $k_n = 3n(2\pi/L)$, or even $\{k_n\} = \{0, 3, 4, 7, 8, \dots\}(2\pi/L)$, for the register modes will ensure that $M(k_j - k_i) = 0$ for all pairs of modes. The

duration of a $k = 2\pi/L$ gradient pulse is $\frac{2\pi\hbar}{m_0\Delta B}$, and to implement such a pulse in 100 ns for the system of Fig. 1, a field gradient of 13 mT/m is required across the sample. A gradient of this strength allows us to sequentially address hundreds of modes, while correspondingly stronger gradients or longer pulse interaction times are needed to switch between any pair of modes.

We have now established that the k modes of the spin ensemble in the strongly polarized limit behave as a large number of independent harmonic oscillators. Using gradient pulses, these spatial modes can be mapped to the $k = 0$ mode which is the only one that can be brought to interact strongly with the cavity field. When the spin ensemble is brought into resonance by adjusting the bias field B , the collective coupling [Eq. (1)] swaps the states of the $k = 0$ mode and the cavity at the effective coupling rate $\sqrt{N}\bar{g}$. We can thus selectively swap the state of the cavity with any k_i mode of the spin ensemble without disturbing any other mode by applying first a $(-k_i)$ pulse followed by a sweep through resonance and finally a (k_i) pulse. To extend our addressable qubit register into a full quantum information processing system, we need to further include a physical qubit system which can be initialized, manipulated, and readout, and which can facilitate a suitably nonlinear interaction between the oscillator modes as outlined in Fig. 2. Our candidate for this qubit is the transmon CPB [6] which has already been integrated experimentally with the system geometry illustrated in Fig. 1. Single qubit unitary operations can be achieved by applying classical resonant fields to the CPB system, and by swapping the states between any selected k_i and the CPB degree of freedom, and one-qubit gates as well as readout [24] can hence be applied to any qubit in our register. By swapping a control qubit to the CPB and subsequently swapping a target qubit to the cavity field, the controlled interaction between these two systems effectively implements a two-qubit gate on any pair of qubits [4].

So far, we have only discussed the coupling of the $k = 0$ mode to a quantum field in a cavity, but the individual addressability and independence of the different k modes

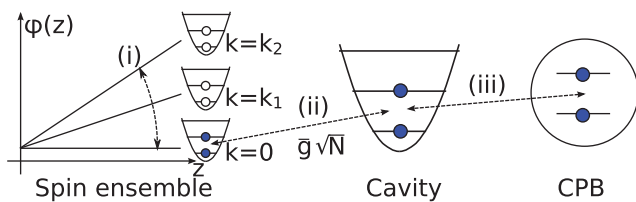


FIG. 2 (color online). Schematic view of the quantum degrees of freedom in our hybrid physical system. The spatial spin modes associated with different k values are indicated by the lines in the $\phi(z)$ vs z diagram (left). (i) A (Δk) gradient pulse interaction maps any mode k to $k' = k + \Delta k$ [Eq. (4)]. (ii) Only the $k = 0$ mode interacts strongly with the cavity field mode [Eq. (1)] which, in turn, is coupled (iii) to the two-level CPB system by a Jaynes-Cummings interaction.

can also be demonstrated with magnetic field gradients and classical resonant field pulses. A short duration classical pulse “tilts” the macroscopic magnetization vector of the sample by a small angle, generating a component of the magnetization perpendicular to z proportional to the tilt angle; the component parallel to z changes only to second order. During a magnetic field gradient pulse, the perpendicular component of the magnetization precesses nonuniformly across the sample, encoding the excitation in a spin wave and reducing the total perpendicular component of the magnetization effectively to zero. Subsequent excitations may be stored in spin wave modes in a similar way. Excitations stored in this way may be detected by applying a gradient pulse that converts a particular spin wave back into a uniform transverse magnetization. In the uniform bias field, the uniform transverse magnetization precesses coherently and its inductive signal may be detected using a standard pulsed magnetic resonance spectrometer.

Our proposal relies on several key assumptions, and we now turn to a discussion of potential errors resulting from mechanisms bringing their validity into question.

In order for the collective spin wave excitations to act as independent oscillator modes, the density of excited spins must be low. This would be adequately fulfilled at an operating temperature of 20 mK where the thermal excitation probability of spins is $p \approx 10^{-5}$. Even at such low temperature, a large total number pN of thermally excited spins will be present in the sample. The reason that we expect to be able to recognize a single collective spin excitation on the background of, possibly, millions of excited spins is that these spin excitations are distributed over all spins and hence over all collective modes of the system, while the excitation in the small number of modes that we interact with is negligible. For each collective mode, the population outside the ground state will only be $\langle \hat{b}^\dagger(k_i)\hat{b}(k_i) \rangle \approx p$. The fact that most of the excitation resides in spectator modes, which are not coupled to the cavity field, can, in the case of homogeneous coupling, be argued more directly in the collective spin representation of the system [25]. Further, the independence of the modes ensures that we may actively cool the k_i modes used in the register by transferring their excitation to the cavity field, allowing an efficient preparation of the ground state of the register, even at finite temperatures.

The dipole-dipole interactions must be weak enough that the spin waves are eigenstates of the system Hamiltonian. This will not be strictly fulfilled, and the dipole-dipole interaction will lead to decoherence of the register. For random doping as considered here, we expect the decoherence rate to be on the order of the line broadening induced by random dipole-dipole coupling [26,27], while for a regular lattice of spins, the dominant decoherence mechanism is expected to be dipole-mediated Raman scattering with thermal spin waves [28,29]. With the spins distributed uniformly in a layer of $1 \mu\text{m}$ in the setup of Fig. 1, the

dipole-induced line broadening would be $\sim 2\pi \times 50$ kHz [30]. Also, in a real experimental apparatus, it is difficult to make the bias field sufficiently homogeneous that different electrons experience the same precession, so that the collective excitation of the ensemble remains coherent for times comparable to the intrinsic dephasing time.

Errors due to static inhomogeneities in the external fields, e.g., field distortions due to the Meissner effect of the superconducting cavity electrodes, are well known in nuclear and electronic spin resonance studies and can be countered by applying a classical refocusing pulse which stimulates a Hahn echo. The superconducting surfaces may also contribute a fluctuating external field [31], the effect of which can be minimized by not positioning spins close to electrode surfaces. As $\sqrt{N}g$ is independent of mode volume for constant spin density, this can be achieved without loss of effective coupling strength. For the system illustrated in Fig. 1, the refocusing could be performed by introducing a strong microwave field at the Larmor frequency, either through the cavity or by an alternative coupling mechanism. Ideally, the refocusing pulses rotate all members of the ensemble by exactly π about an axis perpendicular to z . In practice, however, it is not possible to apply perfect π pulses and this strategy is expected to introduce a very large number of excitations into the system. It might be expected that a significant fraction of such excitations would enter and swamp the register modes, but on closer inspection this is not the case. To argue this, let us for simplicity assume that the classical driving field shares the amplitude and phase characteristics of the cavity mode, $B_0(\mathbf{r})$. In this case, any deviation ε_1 (ε_2) of the area of the first (second) echo pulse from π would introduce $\sim N\varepsilon_2^2$ excitation in the $k = 0$ mode and similarly $\sim N\varepsilon_1^2$ excitations into an “inhomogeneous” mode where the relative phase of the excited spins are given by the inhomogeneous precession over the Hahn pulse interval. In general, the inhomogeneous mode is not a k mode, although in the absence of inhomogeneities it corresponds to the $k = 0$ mode. In the discrete spin case, the mode overlap between any pair of modes is $\sim 1/\sqrt{N}$, so that the Hahn echo errors will introduce $\sim \varepsilon_i^2$ excitations into each register mode. The echo sequence can be applied to free spin ensembles without a cavity and CPB and the effect of the refocusing can thus be demonstrated for classical spin waves. The functioning of the cavity and the CPB will not be seriously impeded, but special means may be needed to avoid excessive direct excitation of these components of the hybrid system during the application of the strong echo pulses.

In summary, we have proposed a quantum register capable of holding hundreds of physical qubits in collective excitations of a spin ensemble, and we have indicated how to perform qubit encoding, one- and two-bit gates, and readout in this system. Further, we have identified mecha-

nisms and properties of the collective states that make the register resilient to the effects of finite polarization and errors introduced by echo sequences used to counter the effects of bias field inhomogeneities.

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